

AP CALCULUS AB STUDY GUIDE

176 High-Yield Concepts + Formulas

Advanced Placement Course | College Board CED-aligned

- All 8 CED units: Limits to Applications of Integration
- Every key rule: power, chain, product, quotient, FTC, u-sub
- Derivative & integral rules, formulas, and worked examples
 - Perfect for the AP Calculus AB exam in May

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How to Use This Guide

This guide condenses the AP Calculus AB course and exam description into high-yield concepts with the core formulas worked into each explanation. It is not a textbook replacement -- it is the fastest way to review everything the exam covers and to target your weak units.

The AP Calculus AB exam is 3 hours 15 minutes: Section I is 45 multiple-choice questions (30 without a calculator in 60 minutes, 15 with a calculator in 45 minutes) and Section II is 6 free-response questions (2 with a calculator, 4 without, in 105 minutes total). The 8 units below map directly to the College Board Course and Exam Description, with unit weights shown on each section.

How to use the concepts

- Each concept = term + concise explanation + high-yield exam tip (+ example where useful).
- First pass: read every concept in order. Underline anything unfamiliar.
- Second pass: focus only on the concepts you underlined.
- Two weeks before the exam: rapid-review every tip line (they are the highest-yield).
- Use the Table of Contents to jump straight to your weakest units.

Table of Contents

1	Unit 1: Limits & Continuity	22 concepts
2	Unit 2: Differentiation: Definition & Basic Rules	22 concepts
3	Unit 3: Composite, Implicit & Inverse Functions	22 concepts
4	Unit 4: Contextual Applications of Differentiation	22 concepts
5	Unit 5: Analytical Applications of Differentiation	22 concepts
6	Unit 6: Integration & Accumulation of Change	22 concepts
7	Unit 7: Differential Equations	22 concepts
8	Unit 8: Applications of Integration	22 concepts

Total: 176 high-yield concepts

How to use the TOC: work through the units in order on your first pass, then jump back to your weakest units for review. Each concept includes an Explanation, a high-yield exam tip, and where useful an Example. Mark the concepts you miss with a small dot in the margin and revisit them 48 hours later.

Unit 1: Unit 1: Limits & Continuity

10-12% of the exam. The foundation of calculus: what happens as x approaches a value.

1	The Limit Concept
Explanation:	The limit of $f(x)$ as x approaches a , written $\lim_{x \rightarrow a} f(x)$, is the value $f(x)$ gets closer to as x gets closer to a (from both sides). Limits describe behavior near a point, not necessarily AT it.
AP Calc Tip:	plug in $x = a$ first. If you get a number, the limit is that number (provided f is defined near a). If you get $0/0$, the limit needs algebraic work.
Example:	$\lim_{x \rightarrow 1} (x^2 - 1)/(x - 1)$ as $x \rightarrow 1$: plugging in gives $0/0$, but factoring gives $\lim_{x \rightarrow 1} (x+1) = 2$.
2	One-Sided Limits
Explanation:	The left-hand limit $\lim_{x \rightarrow a^-} f(x)$ as $x \rightarrow a^-$ considers x approaching a from values less than a ; the right-hand limit $\lim_{x \rightarrow a^+} f(x)$ as $x \rightarrow a^+$ from values greater than a .
AP Calc Tip:	the two-sided limit exists if and only if the left and right limits exist and are EQUAL. Discontinuities often have unequal one-sided limits.
Example:	For $f(x) = x /x$ at $x = 0$: left limit is -1 , right limit is 1 , so lim does not exist.
3	Limits at Infinity (End Behavior)
Explanation:	$\lim_{x \rightarrow +\infty} f(x)$ as $x \rightarrow +\infty$ describes f 's horizontal behavior. For rational functions, compare degrees: if degrees equal, the limit is the ratio of leading coefficients; if numerator degree is larger, the limit is $+\infty$ or $-\infty$; if smaller, the limit is 0 .
AP Calc Tip:	the degree rule for rational limits at infinity is a free point on the FRQ -- memorize the three cases.
Example:	$\lim_{x \rightarrow \infty} (3x^2 + 1)/(2x^2 - 5)$ as $x \rightarrow \infty = 3/2$ (degrees equal, ratio of leading coefficients).
4	Infinite Limits & Vertical Asymptotes
Explanation:	If $f(x)$ grows without bound as $x \rightarrow a$ (e.g., $1/x$ as $x \rightarrow 0^+$), f has an infinite limit at a and $x = a$ is a vertical asymptote. Infinite limits mean the limit does not exist as a number.
AP Calc Tip:	vertical asymptotes occur where a rational function's denominator is 0 AND the numerator is nonzero.
Example:	$f(x) = 1/(x-2)^2$ has a vertical asymptote at $x = 2$; the limit from both sides is $+\infty$.
5	The Squeeze (Sandwich) Theorem
Explanation:	If $g(x) \leq f(x) \leq h(x)$ near a and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} f(x) = L$. Used for limits that resist algebra, like $x \sin(1/x)$.
AP Calc Tip:	the squeeze theorem is the standard tool for $\lim_{x \rightarrow 0} x \sin(1/x)$ as $x \rightarrow 0$ -- bound $ \sin(1/x) \leq 1$.
Example:	Since $- x \leq x \sin(1/x) \leq x $ and both bounds go to 0 , the limit is 0 .
6	Continuity at a Point
Explanation:	f is continuous at $x = a$ if three conditions hold: $f(a)$ is defined, $\lim_{x \rightarrow a} f(x)$ exists, and $\lim_{x \rightarrow a} f(x) = f(a)$. Discontinuities are removable (hole), jump, or infinite.
AP Calc Tip:	check continuity with the three-step test -- all three must hold; a missing value or unequal limit breaks it.
Example:	$f(x) = (x^2-1)/(x-1)$ with $f(1) = 2$ is continuous at $x = 1$ after removing the hole (it equals $x+1$).
7	Types of Discontinuities
Explanation:	Removable: a hole (limit exists, value missing or wrong). Jump: left and right limits differ. Infinite: function blows up (vertical asymptote).
AP Calc Tip:	classify discontinuities by checking the limit: exists = removable; differs by side = jump; unbounded = infinite.
Example:	$f(x) = x /x$ has a jump at 0 (limits -1 and 1); $f(x) = 1/x$ has an infinite discontinuity at 0 .

8 Continuity on an Interval

Explanation: f is continuous on $[a, b]$ if it is continuous at every interior point and one-sided continuous at the endpoints. Polynomials are continuous everywhere; rationals are continuous except where the denominator is 0.

AP Calc Tip: *on the AP exam, continuity on a closed interval is usually given or assumed -- focus on the continuity conditions at isolated points.*

Example: $1/(x^2 - 4)$ is continuous on $(-2, 2)$ since the denominator never vanishes there.

9 The Intermediate Value Theorem (IVT)

Explanation: If f is continuous on $[a, b]$ and k is between $f(a)$ and $f(b)$, then there is at least one c in (a, b) with $f(c) = k$. It guarantees existence, not a value.

AP Calc Tip: *IVT is a justifying phrase on FRQs: name continuity + the interval + the value between endpoints.*

Example: A continuous function from $f(0) = -1$ to $f(1) = 3$ must cross 0 somewhere in $(0, 1)$.

10 Properties of Limits

Explanation: Limits respect arithmetic: $\lim(f+g) = \lim f + \lim g$, $\lim(f \cdot g) = (\lim f)(\lim g)$, $\lim(f/g) = \lim f / \lim g$ (if $\lim g \neq 0$), and $\lim c \cdot f = c \cdot \lim f$. Also $\lim x^n = a^n$ and $\lim e^x = e^a$.

AP Calc Tip: *combine properties to break hard limits into easy pieces; always verify the denominator limit is nonzero before dividing.*

Example: $\lim (3x^2 - 2x + 1)$ as $x \rightarrow 2 = 3(4) - 4 + 1 = 9$ by the sum and power properties.

11 The Limit of Composite Functions

Explanation: If g is continuous at b and $\lim f(x) = b$ as $x \rightarrow a$, then $\lim g(f(x)) = g(b)$ as $x \rightarrow a$. Substitution into a continuous outer function is valid.

AP Calc Tip: *'the limit of a continuous composition is the function applied to the limit' -- a clean justification phrase.*

Example: $\lim \sin(x^2)$ as $x \rightarrow 0 = \sin(\lim x^2) = \sin(0) = 0$ since sine is continuous.

12 Limits Involving Trig Functions

Explanation: The special limit $\lim \sin(x)/x = 1$ as $x \rightarrow 0$ (x in radians) is the cornerstone of trig limits; also $\lim (1 - \cos x)/x = 0$. Derivatives of trig functions rest on these.

AP Calc Tip: *the exam rewards citing $\lim \sin(x)/x = 1$ explicitly -- write it down when you use it.*

Example: $\lim \sin(3x)/x$ as $x \rightarrow 0 = 3 \cdot \lim \sin(3x)/(3x) = 3 \cdot 1 = 3$.

13 Limits of Exponential & Log Functions

Explanation: $\lim e^x = e^a$ as $x \rightarrow a$, $\lim \ln(x) = \ln(a)$ for $a > 0$; as $x \rightarrow \infty$, e^x grows faster than any polynomial (so $e^x/x^n \rightarrow \infty$), and $\ln(x)$ grows slower than any positive power.

AP Calc Tip: *end-behavior comparisons (exponential beats power beats log) are common multiple-choice questions.*

Example: $\lim x/e^x$ as $x \rightarrow \infty = 0$ because the exponential dominates the polynomial.

14 The Definition of Continuity in Terms of Limits

Explanation: f is continuous at a exactly when $\lim f(x) = f(a)$ as $x \rightarrow a$. This single equation encodes all three continuity conditions.

AP Calc Tip: *on justification questions, write 'since $\lim f(x) = f(a)$, f is continuous at a ' -- the one-line proof.*

Example: For $f(x) = x^2$ at $a = 3$: $\lim x^2 = 9 = f(3)$, so f is continuous there.

15 When Limits Fail to Exist

Explanation: A limit fails to exist if: left and right limits differ (jump), the function grows without bound (infinite), or it oscillates without settling (e.g., $\sin(1/x)$ near 0).

AP Calc Tip: *oscillation ($\sin(1/x)$) is the classic 'limit does not exist' trap -- no bound works.*

Example: $\sin(1/x)$ as $x \rightarrow 0$ oscillates between -1 and 1 infinitely fast -- no limit exists.

16 Estimating Limits from Graphs & Tables

Explanation: Given a graph or table, estimate limits by tracking the trend of $f(x)$ as x approaches a . Tables show values approaching a ; graphs show the path of the curve.

AP Calc Tip: *use tables as numerical evidence: if the values settle near a number as x approaches a from both sides, that is the limit estimate.*

Example: A table with $f(1.9)=2.7$, $f(1.99)=2.97$, $f(2.01)=3.03$, $f(2.1)=3.3$ suggests $\lim_{x \rightarrow 2} f(x) = 3$ as $x \rightarrow 2$.

17 Removable Discontinuities & Holes

Explanation: A hole at $x = a$ means $\lim_{x \rightarrow a} f(x)$ exists but $f(a)$ is undefined or off. Holes are 'removable' by redefining $f(a)$ to match the limit.

AP Calc Tip: *factor and cancel to reveal holes: $f(x) = (x-a)g(x)/(x-a)$ reduces to $g(x)$ with a hole at a .*

Example: $f(x) = (x^2 - 4)/(x - 2)$ simplifies to $x + 2$ with a hole at $x = 2$; the limit as $x \rightarrow 2$ is 4.

18 The Formal Definition of Limit (Epsilon-Delta)

Explanation: $\lim_{x \rightarrow a} f(x) = L$ as $x \rightarrow a$ means: for every $\epsilon > 0$ there is a $\delta > 0$ such that $|x - a| < \delta$ implies $|f(x) - L| < \epsilon$. (Conceptual; AP rarely tests epsilon-delta proofs.)

AP Calc Tip: *know the definition conceptually -- 'f(x) can be made arbitrarily close to L by taking x sufficiently close to a.'*

Example: For $f(x) = 2x$ near $x = 1$, choosing $\delta = \epsilon/2$ makes $|2x - 2| < \epsilon$ whenever $|x - 1| < \delta$.

19 Continuity of Composite Functions

Explanation: If f is continuous at a and g is continuous at $f(a)$, then $g(f(x))$ is continuous at a . Compositions preserve continuity.

AP Calc Tip: *justify continuity of composites by citing continuity of each piece in order.*

Example: $\sin(x^2)$ is continuous everywhere because x^2 and sine are both continuous.

20 Using Continuity to Justify Limits

Explanation: If f is continuous at a , then $\lim_{x \rightarrow a} f(x) = f(a)$ -- direct substitution is valid. This is the fastest limit technique on the exam.

AP Calc Tip: *always try direct substitution FIRST; the continuity justification phrase is 'by continuity of f...'*

Example: $\lim_{x \rightarrow e} \ln(x) = \ln(e) = 1$ by continuity of the natural log at e .

21 Limits & Asymptotes of Rational Functions

Explanation: For $f(x) = P(x)/Q(x)$: vertical asymptotes where $Q = 0$ and $P \neq 0$; horizontal asymptote from the degree rule; holes where both vanish (after canceling common factors).

AP Calc Tip: *factor fully, cancel common factors (holes), then locate asymptotes from the reduced form.*

Example: $f(x) = (x-1)/(x^2-1) = 1/(x+1)$ with a hole at $x = 1$ and a vertical asymptote at $x = -1$.

22 The Relationship Between Limits, Derivatives & Continuity

Explanation: Differentiability implies continuity: if f is differentiable at a , then f is continuous at a . But continuity does NOT imply differentiability (e.g., $|x|$ at 0).

AP Calc Tip: *this implication direction (differentiable $\rightarrow$ continuous) is tested constantly; the converse is false.*

Example: $f(x) = |x|$ is continuous at 0 but not differentiable there (corner).

Unit 2: Unit 2: Differentiation: Definition & Basic Rules

10-12% of the exam. The derivative as a rate of change and the basic rules.

23 The Derivative as a Limit

Explanation: The derivative $f'(a) = \lim_{h \rightarrow 0} [f(a+h) - f(a)]/h$ as $h \rightarrow 0$ (or $\lim_{x \rightarrow a} [f(x) - f(a)]/(x - a)$ as $x \rightarrow a$). It is the instantaneous rate of change and the slope of the tangent line.

AP Calc Tip: recognize both limit forms of the derivative on the exam -- they ask you to identify which limit defines $f'(a)$.

Example: For $f(x) = x^2$, $f'(3) = \lim_{h \rightarrow 0} [(3+h)^2 - 9]/h = \lim_{h \rightarrow 0} (6h + h^2)/h = 6$.

24 The Power Rule

Explanation: $d/dx [x^n] = n \cdot x^{n-1}$ for any real n . The workhorse rule of differentiation.

AP Calc Tip: apply the power rule to every term, including fractional and negative exponents: rewrite roots as powers first.

Example: $d/dx [x^5] = 5x^4$; $d/dx [\sqrt{x}] = d/dx [x^{1/2}] = (1/2)x^{-1/2} = 1/(2\sqrt{x})$.

25 The Constant Multiple Rule

Explanation: $d/dx [c \cdot f(x)] = c \cdot f'(x)$. Constants factor out of derivatives.

AP Calc Tip: never drop the constant -- it multiplies the whole derivative.

Example: $d/dx [5x^3] = 5 \cdot 3x^2 = 15x^2$.

26 The Sum & Difference Rules

Explanation: $d/dx [f(x) + g(x)] = f'(x) + g'(x)$ and $d/dx [f(x) - g(x)] = f'(x) - g'(x)$. Derivatives distribute over addition and subtraction.

AP Calc Tip: differentiate term by term -- the sum rule lets you break any polynomial into pieces.

Example: $d/dx [x^3 - 4x^2 + 7] = 3x^2 - 8x$ (the constant 7 has derivative 0).

27 The Derivative of a Constant

Explanation: $d/dx [c] = 0$. The derivative of any constant function is zero -- a horizontal line has slope 0.

AP Calc Tip: constants vanish when differentiating; keep them only when they multiply a variable term.

Example: $d/dx [8] = 0$ and $d/dx [\pi] = 0$.

28 The Product Rule

Explanation: $d/dx [f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$. Differentiate one factor at a time and add.

AP Calc Tip: the product rule is 'first times derivative of second plus second times derivative of first' -- keep the original factors intact.

Example: $d/dx [x^2 \cdot e^x] = 2x \cdot e^x + x^2 \cdot e^x = e^x(x^2 + 2x)$.

29 The Quotient Rule

Explanation: $d/dx [f(x)/g(x)] = [f'(x) \cdot g(x) - f(x) \cdot g'(x)] / [g(x)]^2$. High-low minus low-high, over low squared.

AP Calc Tip: memorize 'lo d hi minus hi d lo, over lo lo' -- order matters (subtraction is not commutative).

Example: $d/dx [x/(x+1)] = [(1)(x+1) - x(1)]/(x+1)^2 = 1/(x+1)^2$.

30 The Chain Rule

Explanation: $d/dx [f(g(x))] = f'(g(x)) \cdot g'(x)$. Differentiate the outside, keep the inside, multiply by the derivative of the inside -- 'derivative of outside, times derivative of inside.'

AP Calc Tip: the chain rule is the most tested differentiation rule -- use it for every composite, including powers of functions.

Example: $d/dx [(x^2 + 1)^3] = 3(x^2 + 1)^2 \cdot 2x = 6x(x^2 + 1)^2$.

31 Derivatives of Exponential Functions

Explanation: $d/dx [e^x] = e^x$ and $d/dx [a^x] = a^x \ln(a)$ for $a > 0$. The exponential function is its own derivative.

AP Calc Tip: for $e^{g(x)}$, the answer is $e^{g(x)} \cdot g'(x)$ (chain rule).

Example: $d/dx [e^{2x}] = 2e^{2x}$; $d/dx [3^x] = 3^x \ln 3$.

32 Derivatives of Natural Logarithms

Explanation: $d/dx [\ln(x)] = 1/x$ for $x > 0$; $d/dx [\ln|x|] = 1/x$ for $x \neq 0$. By the chain rule, $d/dx [\ln(g(x))] = g'(x)/g(x)$.

AP Calc Tip: $\ln$ absolute value differentiates to the same $1/x$ -- the absolute value matters for domains, not the derivative formula.

Example: $d/dx [\ln(5x + 2)] = 5/(5x + 2)$.

33 Derivatives of Sine & Cosine

Explanation: $d/dx [\sin x] = \cos x$ and $d/dx [\cos x] = -\sin x$. All trig derivative formulas follow from these two.

AP Calc Tip: the minus sign belongs to cosine's derivative -- a classic sign-error trap.

Example: $d/dx [\sin(3x)] = 3 \cos(3x)$; $d/dx [\cos(x^2)] = -2x \sin(x^2)$.

34 Derivatives of the Other Trig Functions

Explanation: $d/dx [\tan x] = \sec^2 x$; $d/dx [\cot x] = -\csc^2 x$; $d/dx [\sec x] = \sec x \tan x$; $d/dx [\csc x] = -\csc x \cot x$.

AP Calc Tip: pair them up: $\tan/\sec$ are positive-family, $\cot/\csc$ are negative-family -- memorize the matching pairs.

Example: $d/dx [\tan(2x)] = 2 \sec^2(2x)$; $d/dx [\sec(x^2)] = 2x \sec(x^2) \tan(x^2)$.

35 Derivatives of Inverse Trig Functions

Explanation: $d/dx [\arcsin x] = 1/\sqrt{1-x^2}$; $d/dx [\arctan x] = 1/(1+x^2)$; $d/dx [\operatorname{arcsec} x] = 1/(|x| \sqrt{x^2-1})$.

AP Calc Tip: $\arctan$'s derivative $1/(1+x^2)$ is the one you will actually integrate -- know it both directions.

Example: $d/dx [\arctan(x^2)] = 2x/(1+x^4)$ by the chain rule.

36 Higher-Order Derivatives

Explanation: $f'(x)$ is the derivative of $f(x)$ (second derivative); $f''(x)$ the third, etc. f'' measures concavity and acceleration.

AP Calc Tip: notation alert: f'' vs f^2 means different things -- read carefully on the exam.

Example: $f(x) = x^4$: $f'(x) = 4x^3$, $f''(x) = 12x^2$, $f'''(x) = 24x$, $f^{(4)}(x) = 24$.

37 Differentiability Implies Continuity

Explanation: If f is differentiable at a , then f is continuous at a . The converse fails: continuous functions can have corners, cusps, or vertical tangents where they are not differentiable.

AP Calc Tip: to show f is NOT differentiable at a , show a corner, cusp, vertical tangent, or discontinuity.

Example: $f(x) = |x|$ is continuous at 0 but has a corner there, so it is not differentiable at 0.

38 The Tangent Line

Explanation: The tangent line to $y = f(x)$ at $x = a$ is $y - f(a) = f'(a)(x - a)$. The derivative gives the slope; the point is $(a, f(a))$.

AP Calc Tip: tangent-line problems are a guaranteed FRQ opening: find $f(a)$, $f'(a)$, then write point-slope form.

Example: For $f(x) = x^2$ at $a = 3$: slope is 6, point $(3, 9)$, so $y - 9 = 6(x - 3)$.

39 Instantaneous vs Average Rate of Change

Explanation: The average rate of change on $[a, b]$ is $[f(b) - f(a)]/(b - a)$ (a slope of a secant). The instantaneous rate is $f'(a)$ (the tangent slope) -- the limit of average rates.

AP Calc Tip: average rate = secant slope; instantaneous = tangent slope; the exam distinguishes them explicitly.

Example: For position $s(t)$, the average velocity on $[1, 3]$ is $(s(3) - s(1))/2$, while $v(2) = s'(2)$ is instantaneous velocity.

40 The Derivative as a Function

Explanation: The derivative $f'(x)$ is itself a function that maps each x to the slope of f at that x . Its graph shows where f increases ($f' > 0$), decreases ($f' < 0$), and has extrema ($f' = 0$).

AP Calc Tip: *when given f' graph, read f 's behavior from signs and zeros of f' -- a common FRQ setup.*

Example: If f' is positive on $(0, 3)$ and negative on $(3, 5)$, f increases then decreases with a local max at $x = 3$.

41 Derivatives of General Exponential & Power Functions

Explanation: $d/dx [x^x]$ type functions need logarithmic differentiation: take $\ln$ of both sides, differentiate implicitly. General: $d/dx [f(x)^{g(x)}]$ requires $\ln$ on both sides.

AP Calc Tip: *logarithmic differentiation is the method for variable-base/variable-exponent forms -- set it up explicitly.*

Example: For $y = x^x$: $\ln y = x \ln x$, so $y'/y = \ln x + 1$, giving $y' = x^x (\ln x + 1)$.

42 Logarithmic Differentiation

Explanation: To differentiate $y = [\text{complicated product/quotient/power}]$, take $\ln$ of both sides, use log rules to expand, differentiate implicitly, then multiply by y .

AP Calc Tip: *log differentiation turns products into sums and exponents into coefficients -- the cleanest tool for messy expressions.*

Example: $y = (x^2+1)^5 \sin x$: $\ln y = 5 \ln(x^2+1) + \ln(\sin x)$, differentiate, then solve for y' .

43 Estimating Derivatives from Tables

Explanation: Given a table of values, estimate $f'(a)$ with a symmetric difference quotient: $f'(a) \sim [f(a+h) - f(a-h)]/(2h)$, using the closest available points.

AP Calc Tip: *use the closest symmetric points for the best estimate; the symmetric quotient is the standard AP method.*

Example: With $f(1.8) = 5.2$ and $f(2.2) = 6.8$, $f'(2) \sim (6.8 - 5.2)/(0.4) = 4$.

44 Rates of Change in Context

Explanation: Derivatives model rates: velocity = $s'(t)$, acceleration = $s''(t)$, marginal cost = $C'(x)$, population growth rate = $P'(t)$. Units are 'units of f per unit of x .'

AP Calc Tip: *always state units on context problems -- 'meters per second' earns the interpretation point.*

Example: If $s(t) = t^3 - 6t^2$ meters, then $v(2) = 3(4) - 24 = -12$ m/s (moving left/down at $t = 2$).

Unit 3: Unit 3: Composite, Implicit & Inverse Functions

9-10% of the exam. The chain rule's home: compositions, implicit differentiation, and inverses.

45 The Chain Rule Deep Dive

Explanation: $d/dx [f(g(x))] = f'(g(x)) \cdot g'(x)$. Inside-out: differentiate the outer function, evaluate at the inner function, multiply by the inner derivative.

AP Calc Tip: the chain rule is used in nearly every derivative on the exam -- practice until it is automatic, including with trig, exponential, and log outer functions.

Example: $d/dx [e^{(\sin x)}] = e^{(\sin x)} \cdot \cos x$.

46 Derivatives of Composite Functions

Explanation: Composite derivatives combine rules: product + chain, quotient + chain. Apply rules in layers: outermost operation first.

AP Calc Tip: use the 'outer to inner' order -- for $(x^2 + 1)^3 \cdot e^x$, apply product rule, then chain rule on the power.

Example: $d/dx [(x^2 + 1)^3 \cdot e^x] = 3(x^2 + 1)^2(2x) e^x + (x^2 + 1)^3 e^x$.

47 Implicit Differentiation

Explanation: When y is defined implicitly by an equation (e.g., $x^2 + y^2 = 25$), differentiate both sides with respect to x , treating y as a function of x and appending dy/dx to every y -derivative, then solve for dy/dx .

AP Calc Tip: every time you differentiate a y -term, multiply by dy/dx (chain rule). Then isolate dy/dx .

Example: $x^2 + y^2 = 25$: $2x + 2y(dy/dx) = 0$, so $dy/dx = -x/y$.

48 Implicit Differentiation with Trig & Exp

Explanation: The same method works with trig, exponential, and log terms involving y : each y -term gains dy/dx .

AP Calc Tip: keep dy/dx as a factor and don't distribute it into unrelated terms until you isolate.

Example: $\sin(y) + x = 1$: $\cos(y)(dy/dx) + 1 = 0$, so $dy/dx = -1/\cos(y) = -\sec(y)$.

49 The Derivative of Inverse Functions

Explanation: If $y = f^{-1}(x)$, then $f(y) \cdot dy/dx = 1$, so $(f^{-1})'(x) = 1/f'(f^{-1}(x))$. The derivative of the inverse is the reciprocal of the original derivative at the inverse value.

AP Calc Tip: $(f^{-1})'(a) = 1/f'(b)$ where $b = f^{-1}(a)$ -- find b first, then evaluate f' there.

Example: For $f(x) = x^3 + 1$, $f^{-1}(9) = 2$, so $(f^{-1})'(9) = 1/f'(2) = 1/(3 \cdot 4) = 1/12$.

50 Derivatives of Inverse Trig Functions

Explanation: $d/dx [\arcsin x] = 1/\sqrt{1-x^2}$, $d/dx [\arccos x] = -1/\sqrt{1-x^2}$, $d/dx [\arctan x] = 1/(1+x^2)$, $d/dx [\operatorname{arcsec} x] = 1/(|x| \sqrt{x^2-1})$.

AP Calc Tip: the negative signs belong to $\arccos$ and arccsc -- the 'co' functions.

Example: $d/dx [\arcsin(2x)] = 2/\sqrt{1-4x^2}$.

51 Implicit Differentiation to Find Slopes

Explanation: After implicit differentiation, plug in a point (x_0, y_0) to get the slope of the tangent line at that point.

AP Calc Tip: you need BOTH coordinates -- if only x is given, substitute back into the original equation to find y .

Example: At $(3, 4)$ on $x^2 + y^2 = 25$, $dy/dx = -3/4$, so the tangent is $y - 4 = (-3/4)(x - 3)$.

52 Related Rates Setup

Explanation: Related rates: differentiate an equation linking two changing quantities with respect to TIME t , then substitute known values to find an unknown rate.

AP Calc Tip: write the linking equation FIRST (geometry), then differentiate in t , then plug in -- never plug in before differentiating.

Example: A ladder sliding: $x^2 + y^2 = L^2$; differentiating gives $2x(dx/dt) + 2y(dy/dt) = 0$.

53 The Chain Rule in Related Rates

Explanation: Every variable in a related-rates problem is a function of t , so each differentiation produces a rate term dx/dt , dy/dt , etc.

AP Calc Tip: *label every rate with its sign: decreasing radius means dr/dt is negative.*

Example: If a balloon's radius grows at $dr/dt = 2$ cm/s, the volume changes at $dV/dt = 4\pi r^2 (2)$.

54 Differentiating Inverse Functions at a Point

Explanation: To find $(f^{-1})'(a)$: solve $f(x) = a$ for x (that is b), then compute $1/f'(b)$.

AP Calc Tip: *this is a two-step: first locate the point, then reciprocate the slope.*

Example: For $f(x) = 2x + 3$, $f^{-1}(7) = 2$ and $f'(x) = 2$, so $(f^{-1})'(7) = 1/2$.

55 Logarithmic Differentiation

Explanation: For $y = f(x)^{g(x)}$ or products of many factors: $\ln y = g(x) \ln f(x)$, differentiate implicitly, multiply by y .

AP Calc Tip: *this is the ONLY clean way to differentiate variable-base-variable-exponent functions.*

Example: $y = x^{\sin x}$: $y/y' = \cos x \ln x + \sin x / x$, so $y' = x^{\sin x}(\cos x \ln x + \sin x/x)$.

56 The Derivative of a^x

Explanation: $d/dx [a^x] = a^x \ln a$. Special case $a = e$ gives $d/dx [e^x] = e^x$.

AP Calc Tip: *do not confuse a^x with x^a : the power rule (exponent constant) vs exponential rule (base constant).*

Example: $d/dx [2^x] = 2^x \ln 2$; $d/dx [x^2] = 2x$.

57 The Derivative of $\log_a(x)$

Explanation: $d/dx [\log_a(x)] = 1/(x \ln a)$. Special case: $d/dx [\ln x] = 1/x$.

AP Calc Tip: *convert $\log_a$ to $\ln$ form ($\log_a x = \ln x / \ln a$) if you prefer; the derivative follows.*

Example: $d/dx [\log_{10}(x)] = 1/(x \ln 10)$.

58 Second Derivatives of Implicit Functions

Explanation: To find d^2y/dx^2 implicitly: differentiate dy/dx with respect to x , applying the chain rule to any y in the expression, then substitute the known dy/dx .

AP Calc Tip: *after implicit differentiation, dy/dx usually contains y -- differentiate again keeping dy/dx terms, then substitute.*

Example: From $x^2 + y^2 = 25$ with $dy/dx = -x/y$: $d^2y/dx^2 = -(y - x(dy/dx))/y^2 = -25/y^3$.

59 Composite Function Values from Tables

Explanation: Given tables for f , f' , g , g' : evaluate $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ using the table values.

AP Calc Tip: *chain rule with tables: look up $g(x)$, then f' at $g(x)$, then multiply by $g'(x)$.*

Example: If $g(2) = 5$, $g'(2) = 3$, $f(5) = 7$, then $(f \circ g)'(2) = 7 \cdot 3 = 21$.

60 The Inverse Function Theorem

Explanation: If f is differentiable and strictly monotonic near a with $f'(a) \neq 0$, then f^{-1} is differentiable at $f(a)$ with $(f^{-1})'(f(a)) = 1/f'(a)$.

AP Calc Tip: *$f'(a) = 0$ makes the inverse derivative undefined (vertical tangent) -- check before using the formula.*

Example: For $f(x) = x^3$ at $a = 2$: $(f^{-1})'(8) = 1/(3 \cdot 4) = 1/12$.

61 Higher-Order Chain Rule

Explanation: Repeated chain: $d/dx [\sin(\cos(x^2))] = \cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot 2x$ -- one factor per nesting level.

AP Calc Tip: *peel the layers one at a time; each layer contributes exactly one factor.*

Example: $d/dx [e^{\tan(x^2)}] = e^{\tan(x^2)} \cdot \sec^2(x^2) \cdot 2x$.

62 Implicit Differentiation: Finding Tangents

Explanation: For curves like $x^3 + y^3 = 6xy$, find dy/dx implicitly, then evaluate at the point to write the tangent line.

AP Calc Tip: *keep the curve's equation intact through differentiation -- substitute the point only at the very end.*

Example: $x^3 + y^3 = 6xy$: $3x^2 + 3y^2(dy/dx) = 6y + 6x(dy/dx)$; solve for dy/dx and evaluate.

63 Exponential Models in Derivatives

Explanation: If $y = Ce^{kt}$, then $dy/dt = k * Ce^{kt} = k y$ -- the derivative is proportional to the function itself. This is the model for growth and decay.

AP Calc Tip: *'proportional to the quantity' translates directly to $dy/dt = ky$ -- the exam's favorite modeling statement.*

Example: If a population P satisfies $dP/dt = 0.03P$, it grows at 3% per year: $P = P_0 e^{(0.03t)}$.

64 Local Linearity & Linear Approximations

Explanation: Near $x = a$, $f(x)$ is approximately its tangent line: $f(x) \sim f(a) + f'(a)(x - a)$. This linear approximation is most accurate close to a .

AP Calc Tip: *use the tangent-line approximation for 'estimate $f(2.1)$ given $f(2)$ and $f'(2)$ ' questions.*

Example: With $f(2) = 5$ and $f'(2) = -3$, $f(2.1) \sim 5 - 3(0.1) = 4.7$.

65 Differentiability of Piecewise Functions

Explanation: To check differentiability of a piecewise function at the break point a : check continuity first, then compare the left and right derivatives at a .

AP Calc Tip: *a piecewise function can be continuous but have a corner (different left/right slopes) -- not differentiable.*

Example: $f(x) = x^2$ for $x \leq 1$, $2x - 1$ for $x > 1$: continuous at 1; left slope 2, right slope 2 -- differentiable with $f'(1) = 2$.

66 Domain Considerations for Inverse Trig

Explanation: $\arcsin x$ and $\arccos x$ are defined only on $[-1, 1]$; their derivatives $1/\sqrt{1-x^2}$ inherit that domain. $\arctan$ is defined everywhere.

AP Calc Tip: *state the domain when differentiating inverse trig -- the square root requires $|x| < 1$.*

Example: $d/dx [\arcsin(x^2)] = 2x/\sqrt{1-x^4}$, valid where $|x| < 1$.

Unit 4: Unit 4: Contextual Applications of Differentiation

10-15% of the exam. Rates in context: motion, related rates, and approximations.

67 Interpreting the Derivative in Context

Explanation: If $y = f(x)$, then $f'(a)$ is the instantaneous rate of change of y with respect to x at $x = a$, with units 'units of y per unit of x '.

AP Calc Tip: *interpretation points require units -- write them every time ('gallons per minute').*

Example: If $C(t)$ is cost in dollars at t minutes, $C'(10) = 5$ means cost increases at \$5/min at $t = 10$.

68 Position, Velocity & Acceleration

Explanation: If $s(t)$ is position, $v(t) = s'(t)$ is velocity (with direction) and $a(t) = v'(t) = s''(t)$ is acceleration. Speed = $|v(t)|$.

AP Calc Tip: *velocity sign gives direction (negative = moving left/down); acceleration sign alone does NOT tell speeding up or slowing down.*

Example: $s(t) = t^2 - 4t$: $v(t) = 2t - 4$ (zero at $t = 2$); $a(t) = 2$ (constant).

69 Speeding Up vs Slowing Down

Explanation: A particle speeds up when velocity and acceleration have the SAME sign; slows down when they have OPPOSITE signs.

AP Calc Tip: *compare signs of $v(t)$ and $a(t)$ on each interval -- do not use speed alone.*

Example: If $v = -3$ and $a = -1$ (both negative), the particle moves left and speeds up.

70 Average vs Instantaneous Velocity

Explanation: Average velocity on $[a, b] = (s(b) - s(a))/(b - a)$; instantaneous velocity = $s'(t)$ at a specific time.

AP Calc Tip: *'average velocity' over an interval = secant slope; 'velocity at time t ' = derivative.*

Example: $s(t) = t^2$: average velocity on $[1, 3] = (9-1)/2 = 4$; instantaneous at $t = 2$ is $s'(2) = 4$.

71 Related Rates: Expanding Circles & Spheres

Explanation: For circles ($A = \pi r^2$, $C = 2\pi r$) and spheres ($V = \frac{4}{3}\pi r^3$), differentiate the formula with respect to t .

AP Calc Tip: *write the formula, differentiate in t (chain rule gives dr/dt), then substitute known values.*

Example: If $dr/dt = 3$ cm/s when $r = 5$, then $dA/dt = 2\pi(5)(3) = 30\pi$ cm²/s.

72 Related Rates: Ladder & Shadow Problems

Explanation: Ladders use $x^2 + y^2 = L^2$; shadow problems use similar triangles to relate heights and shadow lengths.

AP Calc Tip: *draw the picture, label rates with signs, and differentiate the geometric relation in t .*

Example: A 10-ft ladder with base moving at 2 ft/s when $x = 6$ ($y = 8$): $2(6)(2) + 2(8)(dy/dt) = 0$ gives $dy/dt = -1.5$ ft/s.

73 Related Rates: Cones & Tanks

Explanation: For a conical tank, $V = \frac{1}{3}\pi r^2 h$; relate r and h by similar triangles before differentiating (r/h constant).

AP Calc Tip: *express r in terms of h (or vice versa) BEFORE differentiating -- this is the key step.*

Example: A cone with $r = h/2$: $V = \frac{1}{3}\pi (h/2)^2 h = \pi h^3/12$, so $dV/dt = \pi h^2/4 (dh/dt)$.

74 Related Rates: Inflating Balloons

Explanation: A spherical balloon: $V = \frac{4}{3}\pi r^3$ and $dV/dt = 4\pi r^2 dr/dt$ -- the surface area times the radius rate.

AP Calc Tip: *solve for dr/dt explicitly and check units (cm/s).*

Example: If $dV/dt = 10$ cm³/s when $r = 2$, then $dr/dt = 10/(4\pi * 4) = 10/(16\pi)$ cm/s.

75 Related Rates: Sliding Ladders (Signs)

Explanation: As the ladder's base slides away, x increases ($dx/dt > 0$) and y decreases ($dy/dt < 0$); the top's speed is negative.

AP Calc Tip: *sign conventions matter for the final answer -- state the direction in your interpretation.*

Example: In the ladder setup, $dy/dt = -1.5$ means the top slides DOWN at 1.5 ft/s.

76 Related Rates: Filling & Draining

Explanation: For any tank, $dV/dt = \text{rate in} - \text{rate out}$. Combine with the geometry of the tank to find the height rate dh/dt .

AP Calc Tip: *the net rate dV/dt drives everything; 'filling at 5 and leaking at 1' means $dV/dt = 4$.*

Example: A tank fills at $3 \text{ ft}^3/\text{min}$ and drains at $1 \text{ ft}^3/\text{min}$: $dV/dt = 2$; with $A = 4 \text{ ft}^2$, $dh/dt = 2/4 = 0.5 \text{ ft/min}$.

77 Linear Approximation & Differentials

Explanation: $f(x) \sim f(a) + f'(a)(x - a)$ near $x = a$. The change in f is approximated by $df = f'(a) dx$ where $dx = x - a$.

AP Calc Tip: *linear approximation problems give $f(a)$ and $f'(a)$ and ask for $f(a + \text{small step})$ -- just plug in.*

Example: $f(2) = 5$, $f'(2) = 3$: $f(2.02) \sim 5 + 3(0.02) = 5.06$.

78 Tangent Line Approximation (Local Linearity)

Explanation: The tangent line at a is the best linear estimate of f nearby; error grows as you move away from a .

AP Calc Tip: *state 'using the tangent line at $x = a$ ' as your method on free-response approximations.*

Example: With $f(1) = 2$ and $f'(1) = 4$, the tangent line $y = 2 + 4(x - 1)$ estimates $f(1.1) = 2.4$.

79 Estimating Change with f'

Explanation: The change in f over $[a, b]$ is approximately $f'(a) * (b - a)$ for small intervals (or via a left/right Riemann-style sum of rates).

AP Calc Tip: *for a table of rates over equal steps, estimate total change by summing rate * step -- a Riemann sum of the derivative.*

Example: With rates 5, 7, 6 (gal/min) over 1-min intervals, total change $\sim 5 + 7 + 6 = 18$ gallons.

80 Interpreting f' Graphs

Explanation: Given the graph of f' , read: f is increasing where $f' > 0$, decreasing where $f' < 0$, and has a local extremum where f' changes sign.

AP Calc Tip: *f' graph questions are common -- always translate the y -values of f' into slopes of f .*

Example: If f' crosses from positive to negative at $x = 4$, f has a local max at $x = 4$.

81 Motion with Velocity Graphs

Explanation: Given $v(t)$: position change over $[a, b]$ is the signed area under v (integral); total distance is the integral of $|v|$; the particle turns where v changes sign.

AP Calc Tip: *for motion problems, area under $v = \text{displacement}$; the exam gives velocity graphs and asks for these areas.*

Example: If $v > 0$ on $[0, 2]$ and $v < 0$ on $[2, 4]$, the particle moves right then left, turning at $t = 2$.

82 Acceleration & Jerk Context

Explanation: $a(t) = v'(t)$ tells how velocity changes; $a(t) > 0$ means velocity is increasing (even if the particle moves left).

AP Calc Tip: *separate 'velocity increasing' ($a > 0$) from 'speeding up' (v and a same sign).*

Example: $v = -5$, $a = +2$: velocity is increasing (less negative) but the particle is slowing down.

83 Rate Models from Tables

Explanation: For tabular rate data, estimate instantaneous rates with symmetric differences and accumulated change with step * rate sums.

AP Calc Tip: *symmetric difference quotient for a rate; left/right/midpoint sums for totals.*

Example: Rates at $t = 0, 1, 2, 3$ of 10, 14, 18, 22: total change over $[0, 3] \sim 14 + 18 + 22 = 54$ (right sum).

84 The Meaning of the Sign of f'

Explanation: $f' > 0$ on an interval: f is increasing. $f' < 0$: f is decreasing. $f' = 0$: horizontal tangent (possible extremum).

AP Calc Tip: *sign analysis of f' is the basis of every increasing/decreasing question.*

Example: $f(x) = (x-1)(x-3)$: $f' > 0$ on $(-\infty, 1)$ and $(3, \infty)$ -- f increases there, decreases on $(1, 3)$.

85 Units of Rate of Change

Explanation: If $y = f(x)$, then f' has units of y-units per x-unit: e.g., dollars per hour, gallons per minute, feet per second squared for acceleration.

AP Calc Tip: *units are a free point on interpretation questions -- write them explicitly.*

Example: If s is in feet and t in seconds, $v = s'$ is in ft/s and $a = s''$ in ft/s².

86 Solving Related Rates: A Procedure

Explanation: 1) Draw and label. 2) Write the geometric equation. 3) Differentiate with respect to t . 4) Substitute known values and rates. 5) Solve for the unknown rate and interpret with units.

AP Calc Tip: *never substitute the given numbers before differentiating -- that freezes the rates.*

Example: The five-step procedure reliably earns full credit on the related-rates FRQ.

87 Related Rates: Two Moving Objects

Explanation: When two objects move (e.g., two cars), relate their positions by the Pythagorean theorem and differentiate: $x^2 + y^2 = d^2$.

AP Calc Tip: *track signs: cars moving apart give positive rates on both x and y .*

Example: Car A goes east at 30, car B north at 40: the distance rate $d' = (x \, dx/dt + y \, dy/dt)/d$ at the moment $x = 3$, $y = 4$ is $(90 + 160)/5 = 50$ mph.

88 Linearization Error

Explanation: The tangent-line estimate errs by an amount proportional to $(x - a)^2$ for well-behaved f ; the error grows with $|x - a|$ and with curvature (f'').

AP Calc Tip: *'is the approximation an over- or underestimate?' -- check concavity: concave up means tangent lies below the curve.*

Example: For $f(x) = x^2$ at $a = 2$, the tangent $y = 4x - 4$ lies below the curve for x near 2 (concave up), so it underestimates f .

Unit 5: Unit 5: Analytical Applications of Differentiation

15-18% of the exam. Extrema, the Mean Value Theorem, and curve analysis.

89 The Mean Value Theorem (MVT)

Explanation: If f is continuous on $[a, b]$ and differentiable on (a, b) , then there is c in (a, b) with $f'(c) = (f(b) - f(a))/(b - a)$. Some tangent slope equals the secant slope.

AP Calc Tip: *justify MVT with BOTH hypotheses (continuity + differentiability) and then set $f'(c)$ = average rate.*

Example: $f(x) = x^2$ on $[1, 3]$: average rate = 4, and $f'(c) = 2c = 4$ gives $c = 2$.

90 Rolle's Theorem

Explanation: A special MVT case: if $f(a) = f(b)$, then $f'(c) = 0$ for some c in (a, b) -- a horizontal secant forces a horizontal tangent.

AP Calc Tip: *Rolle's theorem justifies 'there is a point with zero slope' given equal endpoint values.*

Example: $f(x) = \sin x$ on $[0, \pi]$: $f(0) = f(\pi) = 0$, so $f'(c) = \cos c = 0$ for some c (indeed $c = \pi/2$).

91 Critical Points

Explanation: $x = c$ is a critical point of f if $f'(c) = 0$ or $f'(c)$ does not exist, and c is in f 's domain. Extrema can occur only at critical points or endpoints.

AP Calc Tip: *find candidates for extrema by solving $f' = 0$ AND checking where f' is undefined.*

Example: $f(x) = x^3 - 3x$: $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$, so critical points at $x = 1$ and $x = -1$.

92 The First Derivative Test

Explanation: At a critical point c : if f' changes + to -, f has a local max; if - to +, a local min; if no sign change, neither (stationary inflection).

AP Calc Tip: *build a sign chart for f' around each critical point and read the extrema directly.*

Example: $f' = 3(x-1)(x+1)$: signs - + - around -1 and 1; f has a local max at -1 and local min at 1.

93 The Second Derivative Test

Explanation: If $f''(c) > 0$ at a critical point, f has a local min there; if $f''(c) < 0$, a local max; if $f''(c) = 0$, the test is inconclusive.

AP Calc Tip: *use the second derivative test when f'' is easy to compute; fall back on the first derivative test when $f'' = 0$.*

Example: $f(x) = x^4$: $f'(0) = 0$, test inconclusive, but f clearly has a min at 0 (first derivative test works).

94 Finding Absolute Extrema

Explanation: On a closed interval $[a, b]$, the absolute max/min of a continuous f occur at critical points or endpoints: evaluate f at all candidates and compare.

AP Calc Tip: *this is the closed-interval method: candidates = critical points in (a,b) + endpoints a, b .*

Example: $f(x) = x^2$ on $[-2, 3]$: candidates $x = 0, -2, 3$ give 0, 4, 9 -- absolute max 9 at $x = 3$, min 0 at $x = 0$.

95 Concavity & the Second Derivative

Explanation: f is concave up where $f'' > 0$ (curve opens upward, slopes increasing) and concave down where $f'' < 0$.

AP Calc Tip: *concavity questions ask for intervals where f'' is positive or negative -- from the graph or formula of f'' .*

Example: $f(x) = x^3$: $f'' = 6x > 0$ for $x > 0$ (concave up), < 0 for $x < 0$ (concave down).

96 Points of Inflection

Explanation: A point of inflection is where concavity changes: f'' changes sign (and f is continuous there).

AP Calc Tip: *$f'' = 0$ alone is NOT enough -- the sign must actually change (and the point must be in the domain).*

Example: $f(x) = x^3$ has an inflection at $x = 0$ (f'' changes sign), but $f(x) = x^4$ has none (f'' never changes sign).

97 Increasing & Decreasing Intervals

Explanation: On intervals where $f' > 0$, f increases; where $f' < 0$, f decreases. Use sign charts of f' to identify them.

AP Calc Tip: *report intervals with open parentheses and test points inside each interval on f' .*

Example: $f' = x(x-2)$: $f' > 0$ on $(-\infty, 0)$ and $(2, \infty)$; f decreases on $(0, 2)$.

98 Curve Sketching with Derivatives

Explanation: Combine signs of f' (monotonicity) and f'' (concavity) to sketch f : increasing/decreasing and concave up/down determine the shape.

AP Calc Tip: *mark critical points, inflection candidates, and intercepts before sketching -- the graph must match both sign charts.*

Example: $f' > 0$ and $f'' < 0$: f is increasing and concave down (rising, leveling off) -- the classic learning-curve shape.

99 The Candidate Test / Closed Interval Method

Explanation: To find absolute extrema on $[a, b]$: 1) find critical points, 2) evaluate f at critical points and endpoints, 3) pick the largest and smallest values.

AP Calc Tip: *the endpoint evaluation is mandatory -- extrema can occur at endpoints even when $f' \neq 0$ there.*

Example: $f(x) = \cos x$ on $[0, \pi]$: $f(0) = 1$, $f(\pi) = -1$, $f(\pi/2) = 0$ -- absolute max 1 at $x = 0$, min -1 at $x = \pi$.

100 Optimization: Setting Up

Explanation: Optimization problems: identify the quantity to maximize/minimize, write it as a function of one variable using the constraint, find critical points, and justify with the first or second derivative test.

AP Calc Tip: *the constraint equation lets you eliminate a variable -- write the objective, substitute, then differentiate.*

Example: Maximize area with 100 ft of fence for a rectangle: $A = x(50 - x)$; $A' = 50 - 2x = 0$ gives $x = 25$ (a square).

101 Optimization: Box & Can Problems

Explanation: Open boxes: $V = (L - 2x)(W - 2x)x$ where x is the cut square. Cans: $V = \pi r^2 h$ with surface area constraint.

AP Calc Tip: *draw the diagram, write the volume formula, use the constraint to express V in one variable, then optimize.*

Example: A 12x12 sheet with corner cuts x : $V = x(12-2x)^2$; $V' = (12-2x)(12-6x) = 0$ gives $x = 2$ (max).

102 Optimization: The Fence/Perimeter Problem

Explanation: With a fixed amount of fencing, the rectangle with maximum area is a square. With one side against a wall (3 sides of fence), the optimal width is twice the depth.

AP Calc Tip: *for 3-sided fencing with length L , maximize $A = x(L - 2x)$ -- solve $A' = 0$.*

Example: Fencing 3 sides with 120 ft: $A = x(120 - 2x)$ max at $x = 30$, giving area $30 * 60 = 1800 \text{ ft}^2$.

103 Optimization: Cost & Profit

Explanation: Minimize cost or maximize profit: profit = revenue - cost; optimize at the point where marginal revenue = marginal cost ($MR = MC$).

AP Calc Tip: *the derivative of profit is zero exactly when $R' = C'$ -- the economic first-order condition.*

Example: If $R(x) = 100x - x^2$ and $C(x) = 50 + 20x$, profit $P = 80x - x^2 - 50$, max at $x = 40$.

104 Implicit Optimization (Optimizing with Implicit Relations)

Explanation: When the objective and constraint are related implicitly, use implicit differentiation or substitution to reduce to one variable before optimizing.

AP Calc Tip: *related-rates and optimization often combine: set the derivative to zero at the extreme instant.*

Example: Minimize surface area of a cylinder of fixed volume V : $A = 2\pi r^2 + 2V/r$; $A' = 4\pi r - 2V/r^2 = 0$ gives $r = (V/(2\pi))^{1/3}$.

105 The Graph of f' and Extrema of f

Explanation: From f' 's graph: f has local extrema where f' crosses zero (sign change); f has inflection points where f' has local extrema (f'' changes sign).

AP Calc Tip: *when given f' graph, extrema of $f =$ zeros of f' with sign change; inflection of $f =$ extrema of f' .*

Example: If f' peaks at $x = 2$ (f'' changes + to -), f has an inflection point at $x = 2$.

106 The Graph of f'' and Concavity

Explanation: From f'' 's graph: f is concave up where $f'' > 0$ (graph above x -axis), concave down where $f'' < 0$; inflection where f'' crosses the axis.

AP Calc Tip: *translate f'' graph directly: above axis = concave up.*

Example: If f'' is positive on $(0, 4)$ and negative on $(4, 7)$, f is concave up then down, inflection at $x = 4$.

107 Why the MVT Is Useful

Explanation: The MVT guarantees a point with a given instantaneous rate whenever the average rate exists -- used to prove monotonicity, error bounds, and the Fundamental Theorem.

AP Calc Tip: *MVT conclusions are existence statements: 'there exists c in (a, b) ' -- write exactly that.*

Example: If $f'(x) \geq 2$ everywhere, the MVT implies $f(5) - f(1) \geq 2(4) = 8$.

108 Monotonicity via the Derivative

Explanation: If $f' \geq 0$ on an interval, f is nondecreasing; if $f' \leq 0$, f is nonincreasing; if $f' > 0$ everywhere, f is strictly increasing.

AP Calc Tip: *use f' sign to prove monotonicity on intervals -- a common justification on FRQs.*

Example: $f(x) = x^3$ is strictly increasing everywhere because $f' = 3x^2 \geq 0$ (equal zero only at one point).

109 Extrema on Open Intervals

Explanation: On open intervals, absolute extrema may not exist: $f(x) = x^2$ on $(-1, 1)$ has a min at 0 but no absolute max.

AP Calc Tip: *always check whether the domain is open or closed before claiming an absolute extremum.*

Example: $f(x) = 1/x$ on $(0, 1)$ has no maximum (values grow without bound near 0).

110 Justification Language for Extrema

Explanation: On FRQs, justify: ' f changes from positive to negative at c , so f has a local maximum at c ' or ' $f'(c) < 0$, so by the second derivative test c is a local maximum.'

AP Calc Tip: *name the test you use and state the sign evidence -- graders award the reasoning, not the number.*

Example: The sentence 'since $f''(2) = -6 < 0$, f has a local maximum at $x = 2$ ' earns the justification point.

Unit 6: Unit 6: Integration & Accumulation of Change

17-20% of the exam. Antiderivatives, the Fundamental Theorem, and Riemann sums.

111 Antiderivatives (Indefinite Integrals)

Explanation: An antiderivative of f is a function F with $F' = f$. The indefinite integral is the family of all antiderivatives: $\int f(x) dx = F(x) + C$.

AP Calc Tip: always include $+C$ on indefinite integrals -- dropping it costs a point on every FRQ.

Example: $\int 2x dx = x^2 + C$ because $d/dx [x^2] = 2x$.

112 The Power Rule for Integration

Explanation: $\int x^n dx = x^{(n+1)}/(n+1) + C$ for $n \neq -1$. The inverse of the power rule for differentiation.

AP Calc Tip: for $n = -1$ use the natural log: $\int x^{-1} dx = \ln|x| + C$.

Example: $\int x^3 dx = x^4/4 + C$; $\int 1/x dx = \ln|x| + C$.

113 The Constant Multiple & Sum Rules for Integrals

Explanation: $\int c \cdot f(x) dx = c \cdot \int f(x) dx$ and $\int [f + g] dx = \int f dx + \int g dx$. Integration is linear.

AP Calc Tip: integrate term by term and factor constants out -- mirror the derivative rules.

Example: $\int (3x^2 - 4) dx = x^3 - 4x + C$.

114 Integrating Exponential Functions

Explanation: $\int e^x dx = e^x + C$ and $\int a^x dx = a^x / \ln(a) + C$. For $e^{(kx)}$, divide by k .

AP Calc Tip: $\int e^{(kx)} dx = e^{(kx)}/k + C$ -- the chain-rule reversal.

Example: $\int e^{(3x)} dx = e^{(3x)}/3 + C$; $\int 2^x dx = 2^x / \ln 2 + C$.

115 Integrating Trig Functions

Explanation: $\int \sin x dx = -\cos x + C$; $\int \cos x dx = \sin x + C$; $\int \sec^2 x dx = \tan x + C$; $\int \sec x \tan x dx = \sec x + C$; $\int \csc^2 x dx = -\cot x + C$.

AP Calc Tip: memorize the 'co' negatives: sine's integral and cosecant's and cotangent's have minus signs.

Example: $\int \cos(2x) dx = \sin(2x)/2 + C$.

116 Integrating to the Natural Log & Arctangent

Explanation: $\int 1/x dx = \ln|x| + C$; $\int 1/(1 + x^2) dx = \arctan x + C$; $\int 1/\sqrt{1 - x^2} dx = \arcsin x + C$.

AP Calc Tip: these three 'form' integrals appear as substitution targets -- recognize the denominators.

Example: $\int 1/(1 + x^2) dx = \arctan x + C$, and $\int 1/\sqrt{1 - x^2} dx = \arcsin x + C$.

117 The Definite Integral

Explanation: $\int_a^b f(x) dx$ is the signed area between f and the x -axis from a to b : positive where $f > 0$, negative where $f < 0$.

AP Calc Tip: 'signed area' means regions below the axis subtract -- draw the picture before computing.

Example: $\int_0^2 x dx = 2$ (triangle of base 2, height 2).

118 Riemann Sums (Left, Right, Midpoint)

Explanation: Approximate $\int f dx$ with rectangles: split $[a, b]$ into n equal subintervals of width Δx , use left endpoints, right endpoints, or midpoints to set each rectangle height.

AP Calc Tip: left sum overestimates when f decreases; right sum underestimates; midpoint is usually best.

Example: $f(x) = x^2$ on $[0, 2]$ with $n = 2$: left sum = $0^2 \cdot 1 + 1^2 \cdot 1 = 1$; right sum = $1^2 + 4 = 5$; midpoint = $(0.5)^2 + (1.5)^2 = 2.5$.

119 Trapezoidal Sums

Explanation: The trapezoidal rule averages consecutive rectangle heights: $(\Delta x/2)[f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$.

AP Calc Tip: *trapezoids beat left/right sums in accuracy; the formula pattern is 1, 2, 2, ..., 2, 1 times $\Delta x/2$.*

Example: On $[0, 2]$ with $n = 2$ and $f(x) = x^2$: $(1/2)[0 + 2(1) + 4] = 3$ (exact integral is $8/3 \approx 2.67$).

120 The Fundamental Theorem of Calculus (Part 1)

Explanation: If $F' = f$, then integral from a to b of $f(x) \, dx = F(b) - F(a)$. Definite integrals evaluate via any antiderivative.

AP Calc Tip: *FTC part 1 is how you EVALUATE integrals -- find an antiderivative, then subtract.*

Example: integral from 1 to 3 of $2x \, dx = [x^2]$ from 1 to 3 = $9 - 1 = 8$.

121 The Fundamental Theorem of Calculus (Part 2)

Explanation: If $g(x) = \text{integral from } a \text{ to } x \text{ of } f(t) \, dt$, then $g'(x) = f(x)$. The derivative of an accumulating integral returns the integrand.

AP Calc Tip: *FTC part 2 is the 'derivative of the integral' rule: $d/dx \text{ integral from } a \text{ to } x f(t) \, dt = f(x)$.*

Example: $g(x) = \text{integral from } 0 \text{ to } x \text{ of } \sin(t^2) \, dt$: $g'(x) = \sin(x^2)$.

122 The FTC with Variable Limits (Chain Rule)

Explanation: $d/dx \text{ integral from } a \text{ to } h(x) \text{ of } f(t) \, dt = f(h(x)) * h'(x)$. The upper limit's derivative multiplies the integrand.

AP Calc Tip: *differentiate the upper limit and multiply -- the chain rule applied to FTC part 2.*

Example: $g(x) = \text{integral from } 0 \text{ to } x^2 \text{ of } \cos(t) \, dt$: $g'(x) = \cos(x^2) * 2x$.

123 The FTC with Lower-Limit Functions

Explanation: $d/dx \text{ integral from } g(x) \text{ to } a \text{ of } f(t) \, dt = -f(g(x)) * g'(x)$. Flip the limits to add a negative sign.

AP Calc Tip: *move the variable limit to the top first: split integrals at a constant.*

Example: $d/dx \text{ integral from } x^3 \text{ to } 5 \text{ of } f(t) \, dt = -f(x^3) * 3x^2$.

124 The Accumulation Function

Explanation: $F(x) = \text{integral from } a \text{ to } x \text{ of } f(t) \, dt$ accumulates the net change of f over $[a, x]$; its derivative is $f(x)$, and $F(b) - F(a) = \text{total accumulation}$.

AP Calc Tip: *accumulation functions convert rate data into totals -- the heart of 'rate in/rate out' problems.*

Example: If f is a rate (gal/min), integral from 0 to 10 of $f \, dt$ is the total gallons in 10 minutes.

125 Net Change Theorem

Explanation: integral from a to b of $f'(x) \, dx = f(b) - f(a)$: the integral of a rate of change gives the NET change of the quantity.

AP Calc Tip: *'net change' vs 'total change': integrate f' for net change; integrate $|f'|$ for total change.*

Example: integral from 0 to 5 of $v(t) \, dt = s(5) - s(0)$ -- displacement, not distance.

126 U-Substitution

Explanation: For integral $f(g(x)) * g'(x) \, dx$: let $u = g(x)$, $du = g'(x) \, dx$, substitute, integrate in u , then back-substitute.

AP Calc Tip: *choose u as the 'inside' whose derivative appears as a factor; adjust constants to match du .*

Example: integral $2x \cos(x^2) \, dx$: $u = x^2$, $du = 2x \, dx$, giving $\sin(u) + C = \sin(x^2) + C$.

127 U-Substitution with Definite Integrals

Explanation: Change the limits when substituting: integral from a to b of $f(g(x)) g'(x) \, dx = \text{integral from } g(a) \text{ to } g(b) f(u) \, du$.

AP Calc Tip: *either convert limits (cleanest) or back-substitute and use original limits -- never mix the two.*

Example: integral from 0 to 1 of $2x e^{x^2} \, dx$: $u = x^2$, limits become 0 and 1, giving $e - 1$.

128 The Area Between Two Curves

Explanation: integral from a to b of (top - bottom) dx gives the area between curves, where a and b are intersection points (or given bounds).

AP Calc Tip: draw the region; identify top and bottom functions; integrate top minus bottom.

Example: Between $y = x$ and $y = x^2$ on $[0, 1]$: integral $(x - x^2) dx = 1/2 - 1/3 = 1/6$.

129 Area via Vertical vs Horizontal Slices

Explanation: Use dx slices when the region is bounded by functions of x ; use dy slices (right - left) when bounded by functions of y or when convenient.

AP Calc Tip: choose the slice orientation that needs no splitting -- 'right - left' integrates in y .

Example: For the region bounded by $x = y^2$ and $x = 2 - y$: integrate $(2 - y) - y^2$ in y from -2 to 1 .

130 Average Value of a Function

Explanation: The average value of f on $[a, b]$ is $(1/(b - a))$ integral from a to b of $f(x) dx$ -- the height that gives the same area.

AP Calc Tip: average value = area / width; the MVT for integrals guarantees a point achieving it.

Example: $f(x) = x$ on $[0, 2]$: average = $(1/2)(2) = 1$, achieved at $x = 1$.

131 The Mean Value Theorem for Integrals

Explanation: If f is continuous on $[a, b]$, there is c in $[a, b]$ with $f(c) = (1/(b-a))$ integral $f dx$ -- the average value is attained.

AP Calc Tip: pair with the average value formula for a complete justification.

Example: For $f(x) = x^2$ on $[0, 2]$, average = $4/3$, and $f(c) = 4/3$ at $c = 2/\sqrt{3}$.

132 Even & Odd Function Integrals

Explanation: For even f ($f(-x) = f(x)$): integral from $-a$ to $a = 2 * \text{integral from } 0 \text{ to } a$. For odd f ($f(-x) = -f(x)$): integral from $-a$ to $a = 0$.

AP Calc Tip: symmetric-interval integrals of odd functions vanish -- an instant simplification.

Example: integral from -1 to 1 of $x^3 dx = 0$ (odd); integral from -1 to 1 of $x^2 dx = 2 * \text{integral from } 0 \text{ to } 1 = 2/3$.

Unit 7: Unit 7: Differential Equations

6-12% of the exam. Slope fields, separation of variables, and exponential models.

133 Differential Equations Basics

Explanation: A differential equation relates a function to its derivatives, e.g., $dy/dx = 2x$. A solution is a function satisfying the equation; the general solution has a constant C .

AP Calc Tip: *verify a solution by differentiating and substituting -- the standard check.*

Example: $y = x^2 + C$ solves $dy/dx = 2x$ because $d/dx [x^2 + C] = 2x$.

134 Separable Differential Equations

Explanation: Separable equations can be written as $dy/dx = g(x) h(y)$. Solve by separating variables: put y 's with dy , x 's with dx , then integrate both sides.

AP Calc Tip: *separate FIRST: $dy/h(y) = g(x) dx$, integrate each side, solve for y .*

Example: $dy/dx = x/y$: $y dy = x dx$, so $y^2/2 = x^2/2 + C$, giving $y^2 - x^2 = C$.

135 Separation of Variables with Initial Conditions

Explanation: An initial condition (e.g., $y(0) = 3$) determines C , giving the particular solution.

AP Calc Tip: *apply the initial condition AFTER integrating -- plug in to solve for C exactly.*

Example: $dy/dx = 2x$ with $y(1) = 4$: $y = x^2 + C$, $4 = 1 + C$ so $C = 3$, $y = x^2 + 3$.

136 Slope Fields

Explanation: A slope field draws small line segments of slope dy/dx at grid points, showing the shape of solution curves without solving the equation.

AP Calc Tip: *to match a solution to a slope field, follow the tangent directions -- curves never cross.*

Example: $dy/dx = x$: slopes are negative for $x < 0$ and positive for $x > 0$ -- parabolas $y = x^2/2 + C$.

137 Reading Slope Fields

Explanation: From a slope field, sketch solution curves through given points by tracing the segments; identify equilibrium lines (horizontal segments, $dy/dx = 0$).

AP Calc Tip: *the solution through a point follows the field; equal slopes on vertical lines suggest a function of x alone.*

Example: For $dy/dx = y - 2$, the line $y = 2$ has all horizontal segments -- an equilibrium solution.

138 The Logistic Model (Conceptual)

Explanation: Logistic growth: $dP/dt = kP(1 - P/L)$, where L is the carrying capacity. Growth is fastest at $P = L/2$; P approaches L asymptotically.

AP Calc Tip: *AP Calc AB treats logistic conceptually: know the shape (S-curve), the carrying capacity, and the fastest-growth point.*

Example: A population with carrying capacity 1000 grows fastest at $P = 500$ and levels off at 1000.

139 Exponential Growth & Decay Models

Explanation: $dy/dt = ky$ has solution $y = y_0 e^{k(t)}$: exponential growth for $k > 0$, decay for $k < 0$. Doubling/half-life come from this model.

AP Calc Tip: *'the rate is proportional to the amount' translates to $dy/dt = ky$ -- memorize this translation.*

Example: Radioactive decay with half-life 10 years: $N = N_0 (1/2)^{t/10} = N_0 e^{(-0.0693 t)}$.

140 Solving $dy/dx = ky$

Explanation: Separate: $dy/y = k dt$, integrate: $\ln|y| = kt + C$, exponentiate: $y = Ce^{k(t)}$. The constant C is fixed by an initial value.

AP Calc Tip: *the C from $\ln$ lands in the exponent: $y = e^{k(t+C)} = e^C e^{k(t)}$ -- write the final form $y = C e^{k(t)}$.*

Example: $dy/dt = -0.5y$ with $y(0) = 10$: $y = 10 e^{(-0.5t)}$.

141 Half-Life & Doubling Time

Explanation: Half-life T : $y(T) = y_0/2$ gives $k = -\ln 2 / T$. Doubling time D : $k = \ln 2 / D$.

AP Calc Tip: k and the half-life are linked by $\ln 2$ -- compute one from the other on FRQs.

Example: If a substance halves in 5 years, $k = -\ln(2)/5 \sim -0.1386$ per year.

142 Newton's Law of Cooling

Explanation: $dT/dt = k(T - T_{\text{env}})$: the rate of temperature change is proportional to the difference from the environment. Solutions approach T_{env} exponentially.

AP Calc Tip: define $u = T - T_{\text{env}}$; then $du/dt = k u$ -- an exponential model in disguise.

Example: A 90-degree drink in a 70-degree room cools fastest when hot, slowing as it nears 70.

143 Verifying Solutions

Explanation: To verify y solves a differential equation, differentiate y (using the chain rule/product rule as needed) and substitute into the equation.

AP Calc Tip: verification problems ask 'show that $y = \dots$ satisfies...' -- do the differentiation explicitly.

Example: $y = Ce^{2x}$ solves $y' = 2y$: $y' = 2Ce^{2x} = 2y$. Checked.

144 Matching Slope Fields to Equations

Explanation: Given $dy/dx = f(x, y)$, identify the slope field by testing: where $f = 0$ slopes are flat; where f depends only on x , slopes are constant along vertical lines.

AP Calc Tip: test the equilibrium curve $f = 0$ and a couple of sign regions to match fields.

Example: $dy/dx = x^2 - 1$: slopes zero at $x = 1$ and $x = -1$ (vertical lines of flat segments).

145 The General vs Particular Solution

Explanation: The general solution contains the arbitrary constant C (a family of curves); the particular solution satisfies an initial condition (one curve).

AP Calc Tip: FRQ language: 'find the particular solution $y = f(x)$ ' means use the initial condition.

Example: For $dy/dx = 3x^2$: general $y = x^3 + C$; with $y(2) = 10$, particular $y = x^3 + 2$.

146 Autonomous Differential Equations

Explanation: $dy/dx = f(y)$ (no explicit x) are autonomous: slopes depend only on y ; equilibrium solutions are constant lines where $f(y) = 0$; stability is read from f 's sign.

AP Calc Tip: for $dy/dx = f(y)$, check stability: if $f' > 0$ above the equilibrium and $f' < 0$ below, it is stable (attracting).

Example: $dy/dt = y(3 - y)$: equilibria at $y = 0$ (unstable) and $y = 3$ (stable carrying capacity).

147 Equilibrium Solutions & Stability

Explanation: An equilibrium $y = c$ (where $dy/dx = 0$) is stable if solutions near it return ($f' < 0$ nearby) and unstable if they flee ($f' > 0$ nearby).

AP Calc Tip: classify equilibria with a sign chart of $f(y)$ -- arrows toward the equilibrium mean stable.

Example: For $dP/dt = 0.1P(10 - P)$: $P = 10$ stable, $P = 0$ unstable.

148 Applications: Mixing Problems (Conceptual)

Explanation: Mixing/compartment problems set up $dQ/dt = \text{rate in} - \text{rate out}$; the resulting equation is often separable or exponential in structure.

AP Calc Tip: AP Calc AB expects the setup and solution of simple separable mixing models: $dQ/dt = k(Q_0 - Q)$.

Example: Salt in a tank: $dQ/dt = (\text{concentration in})(\text{flow in}) - (Q/V)(\text{flow out})$ -- separate and integrate.

149 Modeling with Differential Equations

Explanation: To build a model from words: identify the rate (derivative), the proportionality, and any constants; write $dy/dt = \dots$ and solve.

AP Calc Tip: *'proportional to' means a constant times the quantity; 'inversely proportional' means over the quantity.*

Example: A population grows at a rate proportional to its size: $dP/dt = kP$.

150 The Meaning of the Constant of Proportionality

Explanation: In $dy/dt = ky$, k is the continuous growth/decay rate: $k = 0.03$ means 3% per unit time continuously compounded.

AP Calc Tip: *connect k to percent change and to $e^{k(t)}$ for per-period growth.*

Example: $y = 100 e^{(0.03t)}$ grows at a continuous 3% rate; after 1 year, $y \sim 103.05$ (3.05% discrete).

151 Solving Initial Value Problems (IVPs)

Explanation: An IVP is a differential equation plus an initial condition. Solve the DE (separate and integrate), then use the initial condition to find C .

AP Calc Tip: *state the particular solution explicitly and check the initial condition holds.*

Example: $dy/dx = 1/(y)$ with $y(0) = 2$: $y^2/2 = x + 2$, $y = \sqrt{2x + 4}$ (positive branch).

152 Half-Life Problems in Context

Explanation: Carbon dating, drug elimination, and radioactive decay all use $y = y_0 e^{(kt)}$ with k from the half-life.

AP Calc Tip: *set up $y(t) = y_0 (1/2)^{(t/T)}$ directly with half-life T when given, or convert with $\ln 2$.*

Example: Carbon-14 has a half-life of 5730 years: $N = N_0 e^{(-\ln 2/5730 * t)}$.

153 The Exponential Solution's Shape

Explanation: Exponential growth: concave up, doubling over equal intervals, unbounded. Exponential decay: concave up decreasing, approaching the axis asymptotically, halving over equal intervals.

AP Calc Tip: *equal-interval ratios (doubling/halving) are the signature of exponential models.*

Example: If y doubles every 3 years, $y = y_0 * 2^{(t/3)}$ -- check: at $t = 3$, $y = 2y_0$.

154 Approximating Solutions (Conceptual Euler)

Explanation: Euler's method approximates solutions by stepping along tangent lines: $y(n+1) = y(n) + f(x(n), y(n)) * \Delta x$. (AP Calc AB tests one or two steps.)

AP Calc Tip: *one Euler step: new y = old y + slope at the old point times step size -- use the given slope field point.*

Example: $dy/dx = x + y$ from $(0, 1)$ with step 0.5: $y(0.5) \sim 1 + (0 + 1)(0.5) = 1.5$.

Unit 8: Unit 8: Applications of Integration

10-15% of the exam. Area, volume, and motion via accumulation.

155 Average Value of a Function (Applications)

Explanation: Average value on $[a, b] = (1/(b - a)) \int_a^b f(x) dx$. It represents the mean height over the interval.

AP Calc Tip: FRQs ask 'find the average value' -- write the fraction and the integral, evaluate carefully.

Example: $f(x) = \sin x$ on $[0, \pi]$: average $= (1/\pi) \int_0^\pi \sin x dx = 2/\pi$.

156 The Area Between Curves (Review)

Explanation: Area = integral from a to b of (upper - lower) dx with intersection endpoints; for y -functions use (right - left) dy .

AP Calc Tip: sketch first; when curves cross, split the integral at the crossing.

Example: $y = x$ and $y = x^2$ cross at $x = 0, 1$: area = $\int_0^1 (x - x^2) dx = 1/6$.

157 Volumes: The Disc Method

Explanation: Revolving a region around a horizontal axis: $V = \pi \int_a^b [R(x)]^2 dx$, where R is the radius from the axis to the curve.

AP Calc Tip: discs are for rotation around the boundary with no hole; radius = |curve - axis|.

Example: $y = \sqrt{x}$ revolved around the x -axis from 0 to 4: $V = \pi \int_0^4 x dx = 8\pi$.

158 Volumes: The Washer Method

Explanation: When the solid has a hole (region does not touch the axis), $V = \pi \int_a^b [(R_{\text{outer}})^2 - (R_{\text{inner}})^2] dx$.

AP Calc Tip: R_{outer} and R_{inner} are distances from the rotation axis to the far and near curves.

Example: Region between $y = x$ and $y = x^2$ from 0 to 1 revolved around $y = 0$: $V = \pi \int_0^1 (x^2 - x^4) dx = 2\pi/15$.

159 Volumes: The Shell Method

Explanation: For rotation around a vertical axis (or when shells are easier), $V = 2\pi \int_a^b (\text{radius})(\text{height}) dx$, with radius = distance to the axis.

AP Calc Tip: shells integrate parallel to the axis of rotation; radius is horizontal distance for a vertical axis.

Example: $y = x^2$ from 0 to 2 revolved around the y -axis: $V = 2\pi \int_0^2 x(x^2) dx = 8\pi$.

160 Volumes with Known Cross Sections

Explanation: If cross sections perpendicular to the x -axis have area $A(x)$, then $V = \int_a^b A(x) dx$. Common shapes: squares (side^2), semicircles ($\pi/8 * \text{diameter}^2$).

AP Calc Tip: write $A(x)$ in terms of the region's height $h(x) = \text{top} - \text{bottom}$; squares use h^2 , semicircles use $\pi h^2/8$.

Example: Square cross sections with height $h = 2\sqrt{x}$ over $[0, 1]$: $V = \int_0^1 4x dx = 2$.

161 Motion: Displacement vs Distance

Explanation: Displacement over $[a, b] = \int_a^b v(t) dt = s(b) - s(a)$. Total distance = integral of $|v(t)| dt$ -- split at sign changes of v .

AP Calc Tip: total distance requires splitting at every zero of v and flipping signs.

Example: $v(t) = t - 2$ on $[0, 4]$: displacement = $\int_0^4 (t-2) dt = 0$; distance = $\int_0^4 |t-2| dt = 4$.

162 Position from Velocity

Explanation: Given $v(t)$ and initial position $s(a)$: $s(t) = s(a) + \int_a^t v(u) du$ -- the accumulation of velocity.

AP Calc Tip: 'position at time t ' = initial position + net change (accumulated velocity).

Example: $v(t) = 3t^2$ with $s(0) = 5$: $s(t) = 5 + t^3$.

163 Velocity from Acceleration

Explanation: $v(t) = v(a) + \text{integral from } a \text{ to } t \text{ of } a(u) \, du$; then s from v as above. Integrate twice for position.

AP Calc Tip: *each integration adds a constant -- track initial values at each stage.*

Example: $a(t) = 2$ with $v(0) = 1$ and $s(0) = 3$: $v(t) = 1 + 2t$, $s(t) = 3 + t + t^2$.

164 Particle Motion: Turning Points

Explanation: The particle changes direction where v changes sign ($v = 0$ with a sign flip) -- solve $v(t) = 0$ and test the sign of v around each zero.

AP Calc Tip: *a zero of v with no sign change is a pause, not a turn.*

Example: $v(t) = t^2 - 1$: zeros at $t = 1$ and -1 ; on $[0, 3]$, v changes sign at $t = 1$ -- the particle turns there.

165 Speeding Up & Slowing Down (Integration View)

Explanation: Speed = $|v|$; the particle speeds up when v and a have the same sign (using $a = v'$). Analyze signs on each interval between v -zeros.

AP Calc Tip: *compute a from the velocity expression, then compare signs with v .*

Example: $v(t) = t^2 - 4$ on $[0, 5]$: $v < 0$ until 2, $a = 2t > 0$ always; speeding up when v and a agree ($t > 2$).

166 Total Distance from Velocity

Explanation: Total distance = $\text{integral } |v(t)| \, dt$: find all zeros of v , split, integrate with signs, and add absolute areas.

AP Calc Tip: *total distance $\geq$ |displacement| always; equality only if v never changes sign.*

Example: $v(t) = \sin t$ on $[0, 2\pi]$: distance = 4 (two lobes of area 2 each), displacement = 0.

167 Accumulation: Rate In - Rate Out

Explanation: Net change of a quantity = $\text{integral (rate in - rate out) } dt$; the quantity at time t = initial + net accumulation.

AP Calc Tip: *'how much total' problems are accumulation problems -- always integrate the NET rate.*

Example: Water in a tank with inflow 10 and outflow 6 gal/min over 5 min: net gain = $\text{integral } 4 \, dt = 20$ gal.

168 Finding the Amount at Time t

Explanation: $Q(t) = Q(0) + \text{integral from } 0 \text{ to } t \text{ of } Q'(u) \, du$, where Q' is the net rate of change.

AP Calc Tip: *identify the net rate (in - out) before integrating -- reading the problem carefully is half the work.*

Example: If $Q'(t) = 3t^2 + 2$ and $Q(0) = 1$, then $Q(2) = 1 + \text{integral } (3t^2 + 2) \, dt = 1 + 12 = 13$.

169 Max/Min of an Accumulated Quantity

Explanation: The quantity $Q(t)$ is maximized where $Q'(t) = 0$ (net rate crosses from positive to negative); minimized where it crosses negative to positive.

AP Calc Tip: *extrema of accumulations occur where the net rate changes sign -- the rate's zeros matter.*

Example: If net rate $R(t) = t(3 - t)$, Q peaks at $t = 3$ (R goes + to -).

170 The Fundamental Theorem in Motion

Explanation: $s'(t) = v(t)$ and $s''(t) = a(t)$; position is the accumulation of velocity; velocity the accumulation of acceleration.

AP Calc Tip: *chain the FTC: integrate acceleration to get velocity change, velocity to get position change.*

Example: Given $a(t)$, $v(2) = v(0) + \text{integral from } 0 \text{ to } 2 \text{ of } a(t) \, dt$.

171 Average Velocity vs Average Speed

Explanation: Average velocity = displacement / time = $(1/(b-a)) \text{ integral } v \, dt$. Average speed = total distance / time = $(1/(b-a)) \text{ integral } |v| \, dt$.

AP Calc Tip: *the exam distinguishes them explicitly -- read which one is asked.*

Example: If displacement is 0 over 10 s, average velocity is 0, but average speed may be 5 m/s.

172 Setting Up Volume Integrals

Explanation: For each volume problem: identify the axis of rotation, the region, and the slice (disc/washer/shell); write the radius expressions with absolute distances; integrate over the region's bounds.

AP Calc Tip: *draw the region and axis; the radius is always 'distance from axis to curve' -- sign mistakes are the #1 error.*

Example: Revolving $y = x$ around $y = -1$ from 0 to 2: $R = x + 1$, $V = \pi \int_0^2 (x+1)^2 dx$.

173 Cross-Section Shapes Quick Reference

Explanation: Squares: $A = h^2$. Semicircles: $A = \pi h^2/8$ (diameter h). Equilateral triangles: $A = \frac{\sqrt{3}}{4} h^2$. Rectangles of height k : $A = k h$.

AP Calc Tip: *memorize the three standard cross-section area formulas -- they appear verbatim on FRQs.*

Example: With $h(x) = 2x$ over $[0, 1]$, semicircle cross sections: $V = \int_0^1 \pi (4x^2)/8 dx = \pi/6$.

174 The Net Change Theorem in Applications

Explanation: Net change = integral of the rate: temperature change, population change, cost change, and pollution all follow this single idea.

AP Calc Tip: *the theorem is one sentence: the integral of a rate of change over $[a, b]$ equals the total change.*

Example: If a population grows at $P'(t) = 100 e^{-(0.1t)}$ per year, the increase over 10 years is integral from 0 to 10 of P' dt.

175 Interpretation of the Definite Integral

Explanation: integral from a to b of $f(x) dx$ = the accumulated net change/amount: area, distance, volume, total cost, total pollution -- depending on context.

AP Calc Tip: *on interpretation FRQs, name the quantity AND the units of the integral.*

Example: integral from 0 to 60 of $r(t) dt$ with r in gal/min is the total gallons over the hour.

176 Slicing Method General Formula

Explanation: Any volume = integral of cross-sectional area: $V = \int_a^b A(x) dx$, where $A(x)$ is the area of the slice perpendicular to the x -axis at position x .

AP Calc Tip: *the general slicing formula unifies discs, washers, and custom cross sections -- always start with $V = \int A dx$.*

Example: For discs, $A = \pi R^2$; for squares, $A = h^2$; for triangles, $A = bh/2$ -- plug in and integrate.

What's Next?

You have now reviewed every unit of the AP Calculus AB course. The concepts that matter most are the ones you almost knew -- go back to the Table of Contents, find the units you marked, and reread just those tip lines. Two weeks out, do a rapid pass over the entire guide; the day before, review only the tip lines and the formulas. On free-response questions, write the setup (limits, integrals, derivatives) before computing -- setup earns most of the credit, and always include $+C$ on indefinite integrals and units on interpretation questions.

Pair it with the Study Mastery Cheat Sheet (\$19) -- how to actually retain all of it

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