

AP CALCULUS BC STUDY GUIDE

182 High-Yield Concepts + Formulas

Advanced Placement Course | College Board CED-aligned

- All 10 CED units: Limits & Continuity through Infinite Sequences & Series
- Every AB topic (limits, derivatives, integrals, applications) plus all BC-only topics
- BC exclusives: parametric/polar/vector functions, Euler's method, integration by parts, partial fractions, improper integrals
- Sequences & series: convergence tests, power series, Taylor/Maclaurin polynomials, Lagrange error bound
- Real exam format: 3 hr 15 min, 45 multiple choice + 6 free response

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How to Use This Guide

This guide condenses the AP Calculus BC course and exam description into high-yield concepts. It is not a textbook replacement -- it is the fastest way to review everything the exam covers and to target your weak skills.

The AP Calculus BC exam is 3 hours 15 minutes: Section I has 45 multiple-choice questions (105 minutes, 50% of the score) -- Part A is 30 questions with no calculator, Part B is 15 questions with a graphing calculator. Section II has 6 free-response questions (90 minutes, 50% of the score) -- Part A is 2 questions with a calculator, Part B is 4 questions without one. The 10 units below map to the College Board Course and Exam Description.

How to use the concepts

- Each concept = term + concise explanation + high-yield exam tip (+ example where useful).
- First pass: read every concept in order. Underline anything unfamiliar.
- Second pass: focus only on the concepts you underlined.
- Two weeks before the exam: rapid-review every tip line (they are the highest-yield).
- Use the Table of Contents to jump straight to your weakest units.

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Total: 182 high-yield concepts

How to use the TOC: work through the units in order on your first pass, then jump back to your weakest units for review. Each concept includes an Explanation, a high-yield exam tip, and where useful an Example. Mark the concepts you miss with a small dot in the margin and revisit them 48 hours later.

Unit 1: Unit 1: Limits & Continuity

~4% of the exam. The foundation of calculus: limits, continuity, and asymptotes (reviewed from AB, tested at BC depth).

1 The Limit Concept

Explanation: $\lim f(x)$ as $x \rightarrow a$ is the value $f(x)$ approaches as x gets arbitrarily close to a from both sides. Limits describe behavior NEAR a , not necessarily AT a .

AP Calc BC Tip: always try direct substitution first; if you get a number, that is the limit (when f is defined near a).

Example: $\lim (x^2 - 1)/(x - 1)$ as $x \rightarrow 1$: direct substitution gives $0/0$, but factoring gives $\lim (x+1) = 2$.

2 One-Sided Limits

Explanation: The left-hand limit ($x \rightarrow a^-$) and right-hand limit ($x \rightarrow a^+$) must BOTH exist and be EQUAL for the two-sided limit to exist.

AP Calc BC Tip: jump discontinuities and piecewise functions are where one-sided limits diverge -- check both sides.

Example: For $f(x) = |x|/x$ at 0 : left limit -1 , right limit $+1$, so the limit does not exist.

3 Limits at Infinity

Explanation: For rational functions, compare degrees: equal degrees give the ratio of leading coefficients; numerator larger gives $+\infty/-\infty$; denominator larger gives 0 .

AP Calc BC Tip: the degree rule is a free point on the exam -- memorize all three cases.

Example: $\lim (4x^2 + 3)/(2x^2 - 1)$ as $x \rightarrow \infty = 4/2 = 2$.

4 Infinite Limits & Vertical Asymptotes

Explanation: If $f(x)$ grows without bound as $x \rightarrow a$, the limit is infinite and $x = a$ is a vertical asymptote. Infinite limits are not finite limits.

AP Calc BC Tip: vertical asymptotes occur where the denominator is 0 and the numerator is nonzero after simplification.

Example: $f(x) = 1/(x - 3)^2$ has a vertical asymptote at $x = 3$ with limit $+\infty$ from both sides.

5 The Squeeze Theorem

Explanation: If $g(x) \leq f(x) \leq h(x)$ near a and $\lim g = \lim h = L$, then $\lim f = L$. Used for limits that resist algebra.

AP Calc BC Tip: squeeze is the standard tool for $\lim x \sin(1/x)$ as $x \rightarrow 0$; bound $|\sin(1/x)| \leq 1$.

Example: Since $-|x| \leq x \sin(1/x) \leq |x|$ and both bounds tend to 0 , the limit is 0 .

6 Continuity at a Point

Explanation: f is continuous at a if $f(a)$ is defined, $\lim f(x)$ exists, and $\lim f(x) = f(a)$.

AP Calc BC Tip: the three-part test must ALL pass; a missing value or mismatched limit breaks continuity.

Example: $f(x) = (x^2 - 1)/(x - 1)$ with $f(1) = 2$ is continuous at 1 because the hole is filled.

7 Types of Discontinuities

Explanation: Removable (hole: limit exists), jump (one-sided limits differ), and infinite (vertical asymptote: unbounded).

AP Calc BC Tip: classify by the limit -- exists = removable, differs = jump, unbounded = infinite.

Example: $|x|/x$ has a jump at 0 ; $1/x$ has an infinite discontinuity at 0 ; $(x^2 - 1)/(x - 1)$ has a removable hole at 1 .

8 The Intermediate Value Theorem

Explanation: If f is continuous on $[a, b]$ and k lies between $f(a)$ and $f(b)$, then $f(c) = k$ for some c in (a, b) . IVT guarantees existence.

AP Calc BC Tip: name continuity, the interval, and the value between endpoints when justifying with IVT.

Example: A continuous f with $f(0) = -1$ and $f(1) = 3$ must cross zero somewhere in $(0, 1)$.

9 Properties of Limits

Explanation: $\lim(f+g) = \lim f + \lim g$, $\lim(f \cdot g) = (\lim f)(\lim g)$, $\lim(f/g) = \lim f / \lim g$ (when $\lim g \neq 0$), $\lim c \cdot f = c \cdot \lim f$.

AP Calc BC Tip: *verify the denominator limit is nonzero before applying the quotient property.*

Example: $\lim (3x^2 - 2x)$ as $x \rightarrow 2 = 3(4) - 4 = 8$ by sum and power properties.

10 Limits of Composite Functions

Explanation: If g is continuous at b and $\lim f(x) = b$, then $\lim g(f(x)) = g(b)$.

AP Calc BC Tip: *'the limit of a continuous composition is the function applied to the limit' -- a clean justification phrase.*

Example: $\lim \sin(x^2)$ as $x \rightarrow 0 = \sin(0) = 0$ since sine is continuous.

11 Special Trig Limits

Explanation: $\lim \sin(x)/x = 1$ and $\lim (1 - \cos x)/x = 0$ as $x \rightarrow 0$ (x in radians). These anchor every trig derivative.

AP Calc BC Tip: *write out $\lim \sin(x)/x = 1$ explicitly when you use it -- the exam rewards the citation.*

Example: $\lim \sin(3x)/x$ as $x \rightarrow 0 = 3 \cdot \lim \sin(3x)/(3x) = 3$.

12 L'Hopital's Rule

Explanation: If $\lim f/g$ is $0/0$ or ∞/∞ , then $\lim f/g = \lim f'/g'$ (provided the latter limit exists or is infinite). BC-only formal tool, also usable in units 4-6.

AP Calc BC Tip: *check the indeterminate form FIRST ($0/0$ or ∞/∞); never apply L'Hopital to a determinate limit.*

Example: $\lim (e^x - 1)/x$ as $x \rightarrow 0 = \lim e^x/1 = 1$.

13 L'Hopital with Repeated Application

Explanation: Apply L'Hopital repeatedly while the quotient stays $0/0$ or ∞/∞ ; stop as soon as the form resolves.

AP Calc BC Tip: *after each application, re-check the form; applying it to a resolved limit gives a wrong answer.*

Example: $\lim x^2/e^x$ as $x \rightarrow \infty$: apply twice to get $\lim 2/e^x = 0$.

14 L'Hopital for $\infty - \infty$ and $0 \cdot \infty$

Explanation: Rewrite products ($f \cdot g = f/(1/g)$) and differences (common denominator, factoring, or conjugates) into $0/0$ or ∞/∞ before applying L'Hopital.

AP Calc BC Tip: *for $0 \cdot \infty$, move the simpler factor into the denominator; for $\infty - \infty$, combine or factor.*

Example: $\lim x \ln x$ as $x \rightarrow 0^+$: rewrite as $\ln x / (1/x)$, then L'Hopital gives $\lim (1/x)/(-1/x^2) = \lim -x = 0$.

15 Limits of Exponential & Log Functions

Explanation: $\lim e^x = e^a$, $\lim \ln x = \ln a$ ($a > 0$). As $x \rightarrow \infty$: exponential beats any polynomial, which beats any logarithm.

AP Calc BC Tip: *end-behavior comparisons are frequent multiple-choice questions.*

Example: $\lim x/e^x$ as $x \rightarrow \infty = 0$ because the exponential dominates.

16 Estimating Limits from Graphs & Tables

Explanation: Track $f(x)$ values as x approaches a from both sides; tables and graphs provide numerical evidence for a limit estimate.

AP Calc BC Tip: *use the closest symmetric points in a table for the best estimate.*

Example: $f(1.9)=2.7$, $f(1.99)=2.97$, $f(2.01)=3.03$, $f(2.1)=3.3$ suggests $\lim f(x) = 3$ as $x \rightarrow 2$.

17 Continuity of Composites

Explanation: If f is continuous at a and g is continuous at $f(a)$, then $g(f(x))$ is continuous at a .

AP Calc BC Tip: *justify composite continuity by citing each piece's continuity in order.*

Example: $\sin(x^2)$ is continuous everywhere since x^2 and sine are continuous.

Explanation: Differentiability implies continuity, but continuity does not imply differentiability (corners, cusps, vertical tangents).

AP Calc BC Tip: *to show non-differentiability at a , point to a corner, cusp, or vertical tangent.*

Example: $f(x) = |x|$ is continuous at 0 but not differentiable there (corner).

Unit 2: Unit 2: Differentiation: Definition & Fundamental Properties

~4% of the exam. The derivative definition and the basic differentiation rules.

19 The Derivative as a Limit

Explanation: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$. Instantaneous rate of change and tangent slope.

AP Calc BC Tip: recognize both limit forms of the derivative; the exam asks which limit defines $f'(a)$.

Example: For $f(x) = x^2$, $f'(3) = \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h} = 6$.

20 The Power Rule

Explanation: $\frac{d}{dx} [x^n] = n x^{n-1}$ for any real n . Rewrite roots and reciprocals as powers first.

AP Calc BC Tip: apply the power rule to fractional and negative exponents too.

Example: $\frac{d}{dx} [\sqrt{x}] = \frac{d}{dx} [x^{1/2}] = (1/2)x^{-1/2} = 1/(2 \sqrt{x})$.

21 Constant Multiple & Sum Rules

Explanation: $\frac{d}{dx} [c f] = c f'$ and $\frac{d}{dx} [f + g] = f' + g'$. Derivatives are linear: they distribute over sums and constants.

AP Calc BC Tip: differentiate term by term; constants vanish unless they multiply a variable term.

Example: $\frac{d}{dx} [3x^4 - 2x^2 + 5] = 12x^3 - 4x$.

22 The Product Rule

Explanation: $\frac{d}{dx} [f g] = f' g + f g'$. One factor at a time, then add.

AP Calc BC Tip: keep the original factors intact: 'first times derivative of second plus second times derivative of first.'

Example: $\frac{d}{dx} [x^2 e^x] = 2x e^x + x^2 e^x$.

23 The Quotient Rule

Explanation: $\frac{d}{dx} \left[\frac{f}{g} \right] = \frac{(f' g - f g')}{g^2}$. 'Lo d hi minus hi d lo, over lo lo.'

AP Calc BC Tip: the subtraction order matters -- write $(f' g - f g')$ exactly.

Example: $\frac{d}{dx} \left[\frac{x}{(x+1)} \right] = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$.

24 The Chain Rule

Explanation: $\frac{d}{dx} [f(g(x))] = f'(g(x)) g'(x)$. Outside derivative, keep inside, times inside derivative.

AP Calc BC Tip: the chain rule appears in nearly every BC derivative -- practice until automatic.

Example: $\frac{d}{dx} [\sin(x^2)] = 2x \cos(x^2)$.

25 Exponential & Log Derivatives

Explanation: $\frac{d}{dx} [e^x] = e^x$; $\frac{d}{dx} [a^x] = a^x \ln a$; $\frac{d}{dx} [\ln x] = 1/x$; $\frac{d}{dx} [\ln(g(x))] = g'(x)/g(x)$.

AP Calc BC Tip: for $e^{g(x)}$, the answer includes $g'(x)$; for $\ln(g(x))$, the answer is g'/g .

Example: $\frac{d}{dx} [e^{3x}] = 3e^{3x}$; $\frac{d}{dx} [\ln(5x)] = 5/(5x) = 1/x$.

26 Trig Derivatives

Explanation: $\frac{d}{dx} [\sin x] = \cos x$; $\frac{d}{dx} [\cos x] = -\sin x$; $\frac{d}{dx} [\tan x] = \sec^2 x$; $\frac{d}{dx} [\sec x] = \sec x \tan x$; $\frac{d}{dx} [\cot x] = -\csc^2 x$; $\frac{d}{dx} [\csc x] = -\csc x \cot x$.

AP Calc BC Tip: the minus signs live with cosine, cotangent, and cosecant.

Example: $\frac{d}{dx} [\tan(2x)] = 2 \sec^2(2x)$.

27 Inverse Trig Derivatives

Explanation: $d/dx [\arcsin x] = 1/\sqrt{1-x^2}$; $d/dx [\arctan x] = 1/(1+x^2)$; $d/dx [\operatorname{arcsec} x] = 1/(|x|\sqrt{x^2-1})$.

AP Calc BC Tip: *arctan's derivative $1/(1+x^2)$ matters in BOTH directions -- integrate it later in Unit 6.*

Example: $d/dx [\arctan(x^2)] = 2x/(1+x^4)$.

28 Higher-Order Derivatives

Explanation: f' is the derivative of f ; f'''' is the fourth derivative. f'' gives concavity and acceleration.

AP Calc BC Tip: *in Unit 10, Taylor series use the n th derivative at a point -- practice evaluating $f^{(n)}(a)$.*

Example: $f(x) = x^4$: $f''''(x) = 24$.

29 Differentiability Implies Continuity

Explanation: If f is differentiable at a , then f is continuous at a . The converse is false.

AP Calc BC Tip: *cite the direction carefully -- differentiable $\rightarrow$ continuous, never the reverse.*

Example: $f(x) = |x|$ is continuous but not differentiable at 0.

30 The Tangent Line

Explanation: The tangent line at $x = a$: $y - f(a) = f'(a)(x - a)$.

AP Calc BC Tip: *tangent lines open many FRQs -- find $f(a)$, $f'(a)$, then write point-slope form.*

Example: For $f(x) = x^2$ at $a = 3$: $y - 9 = 6(x - 3)$.

31 Average vs Instantaneous Rate

Explanation: Average rate on $[a, b] = [f(b) - f(a)]/(b - a)$ (secant slope); instantaneous rate = $f'(a)$ (tangent slope).

AP Calc BC Tip: *average = secant, instantaneous = tangent; label units in context problems.*

Example: For position $s(t)$, average velocity on $[1, 3]$ is $(s(3) - s(1))/2$; $v(2) = s'(2)$ is instantaneous.

32 The Derivative as a Function

Explanation: $f'(x)$ maps each x to the slope of f there. $f' > 0$ means f increasing; $f' = 0$ marks candidate extrema.

AP Calc BC Tip: *given f' , read f 's behavior from the signs and zeros of f' .*

Example: If $f' > 0$ on $(0, 3)$ and $f' < 0$ on $(3, 5)$, f has a local max at $x = 3$.

33 Estimating Derivatives from Tables

Explanation: Use the symmetric difference quotient $f'(a) \sim [f(a+h) - f(a-h)]/(2h)$ with the closest available points.

AP Calc BC Tip: *the symmetric quotient is the standard AP method for table-based derivatives.*

Example: With $f(1.8) = 5.2$ and $f(2.2) = 6.8$, $f'(2) \sim 6.8 - 5.2$ over $0.4 = 4$.

34 Logarithmic Differentiation

Explanation: Take $\ln$ of both sides, expand with log rules, differentiate implicitly, solve for y' .

AP Calc BC Tip: *use log differentiation for variable-exponent forms like x^x and messy products.*

Example: $y = x^x$: $\ln y = x \ln x$, so $y' = x^x (\ln x + 1)$.

35 Rates of Change in Context

Explanation: Velocity = $s'(t)$, acceleration = $s''(t)$, marginal cost = $C'(x)$, population growth = $P'(t)$. Units: f -units per x -unit.

AP Calc BC Tip: *always state units on context problems -- that earns the interpretation point.*

Example: If $s(t) = t^3 - 6t^2$ meters, then $v(2) = -12$ m/s (moving left at $t = 2$).

Explanation: The derivative exists when the difference quotient's limit exists; left and right difference quotients must agree.

AP Calc BC Tip: *piecewise functions at boundaries -- check both one-sided difference quotients.*

Example: For $f(x) = |x|$ at 0, the left quotient gives -1 and the right gives +1, so $f'(0)$ does not exist.

Unit 3: Unit 3: Differentiation: Composite, Implicit & Inverse Functions

~4% of the exam. Chain rule applications, implicit differentiation, and inverse functions.

37 Implicit Differentiation

Explanation: Differentiate both sides of an equation with respect to x , treating y as a function of x (multiply by dy/dx), then solve for dy/dx .

AP Calc BC Tip: every term with y gets a dy/dx factor via the chain rule.

Example: $x^2 + y^2 = 25$ gives $2x + 2y \, dy/dx = 0$, so $dy/dx = -x/y$.

38 Derivatives of Inverse Functions

Explanation: If f is invertible and differentiable at a with $f'(a) \neq 0$, then $(f^{-1})'(f(a)) = 1/f'(a)$.

AP Calc BC Tip: $(f^{-1})'$ at a point = 1 over f' at the corresponding point -- swap x and y first.

Example: If $f(2) = 5$ and $f'(2) = 3$, then $(f^{-1})'(5) = 1/3$.

39 The Chain Rule with Multiple Layers

Explanation: For $h(x) = f(g(k(x)))$, $h'(x) = f'(g(k(x))) \, g'(k(x)) \, k'(x)$. Unwind the composition from outside in.

AP Calc BC Tip: count the layers; each nested function contributes exactly one derivative factor.

Example: $d/dx [\sin^3(x^2)] = 3 \sin^2(x^2) \cdot \cos(x^2) \cdot 2x$.

40 Related Rates Setup

Explanation: Related rates: two or more quantities change together. Differentiate the linking equation with respect to TIME t , then substitute known values.

AP Calc BC Tip: draw the picture, name the variables, find the linking equation, then differentiate implicitly in t .

Example: A ladder sliding: $x^2 + y^2 = L^2$ gives $2x \, dx/dt + 2y \, dy/dt = 0$.

41 Related Rates: Geometry

Explanation: Common linking equations: volume of a sphere $V = (4/3)\pi r^3$, cone $V = (1/3)\pi r^2 h$, Pythagorean relations, similar triangles.

AP Calc BC Tip: similar triangles give a relation between r and h in cone problems -- use it to eliminate a variable.

Example: For a cone, r/h constant from similar triangles lets you write V as a function of h alone.

42 Second Derivatives of Parametric Functions

Explanation: For $x(t)$, $y(t)$: $dy/dx = (dy/dt)/(dx/dt)$; $d^2y/dx^2 = d/dt(dy/dx) / (dx/dt)$ -- NOT $(d^2y/dt^2)/(d^2x/dt^2)$.

AP Calc BC Tip: the second parametric derivative formula is BC-specific and frequently tested.

Example: For $x = t^2$, $y = t^3$: $dy/dx = (3t^2)/(2t) = (3/2)t$; $d^2y/dx^2 = (3/2)/(2t) = 3/(4t)$.

43 The Derivative of an Inverse Function at a Point

Explanation: To find $(f^{-1})'(b)$: find a such that $f(a) = b$, then $(f^{-1})'(b) = 1/f'(a)$.

AP Calc BC Tip: swap x and y coordinates; the inverse's slope is the reciprocal of the original's slope.

Example: If $f(4) = 7$ and $f'(4) = 5$, then $(f^{-1})'(7) = 1/5$.

44 Differentiating Piecewise Functions

Explanation: A piecewise function is differentiable at a boundary if it is continuous there AND the left/right derivatives match.

AP Calc BC Tip: check continuity first; a corner (mismatched slopes) fails differentiability.

Example: For f to be differentiable at the join of pieces, both values AND both one-sided derivatives must agree.

45 The Chain Rule in Related Rates

Explanation: Rates chain through compositions: $dV/dt = dV/dr \cdot dr/dt$ when V depends on r which depends on t .

AP Calc BC Tip: *chain rates together -- intermediate variables carry the time derivative.*

Example: $dV/dt = 4\pi r^2 dr/dt$ for a sphere's volume.

46 Inverse Trig with Chain Rule

Explanation: Combine inverse trig derivatives with the chain rule: $d/dx [\arcsin(g(x))] = g'(x)/\sqrt{1 - g(x)^2}$.

AP Calc BC Tip: *the derivative formula requires $|g(x)| < 1$; the chain factor $g'(x)$ is the common error spot.*

Example: $d/dx [\arcsin(2x)] = 2/\sqrt{1 - 4x^2}$.

47 Hyperbolic Functions (BC Plus)

Explanation: $d/dx [\sinh x] = \cosh x$; $d/dx [\cosh x] = \sinh x$; $d/dx [\tanh x] = \text{sech}^2 x$. Hyperbolic functions appear in BC-only contexts.

AP Calc BC Tip: *hyperbolic derivatives have NO sign flips -- unlike trig, $\cosh'$ is $+\sinh$.*

Example: $d/dx [\sinh(2x)] = 2 \cosh(2x)$.

48 The Mean Value Theorem

Explanation: If f is continuous on $[a, b]$ and differentiable on (a, b) , then $f'(c) = [f(b) - f(a)]/(b - a)$ for some c in (a, b) .

AP Calc BC Tip: *MVT justifies that average rate equals instantaneous rate somewhere; Rolle's Theorem is the $f(a) = f(b)$ case.*

Example: A car averaging 60 mph must have hit exactly 60 mph at some instant.

49 The Extreme Value Theorem

Explanation: A continuous function on a closed interval $[a, b]$ attains both a maximum and a minimum on that interval.

AP Calc BC Tip: *candidates for extrema on $[a, b]$ are critical points AND endpoints.*

Example: $f(x) = x^2$ on $[-2, 1]$ has max at $x = -2$ and min at $x = 0$ (critical point).

50 Rolle's Theorem

Explanation: If f is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then $f'(c) = 0$ for some c in (a, b) .

AP Calc BC Tip: *Rolle is MVT with a zero numerator -- name it explicitly when $f(a) = f(b)$.*

Example: $f(x) = \sin x$ on $[0, 2\pi]$: $f(0) = f(2\pi) = 0$, so $f'(c) = 0$ somewhere ($c = \pi$).

51 Implicit Differentiation of Second Derivatives

Explanation: Differentiate dy/dx implicitly AGAIN to get d^2y/dx^2 , substituting dy/dx where it appears.

AP Calc BC Tip: *keep dy/dx as a variable and substitute its expression at the very end.*

Example: $x^2 + y^2 = 25$: $dy/dx = -x/y$; differentiating again gives $d^2y/dx^2 = -25/y^3$.

52 Derivatives of Inverse Trig: Proof Pattern

Explanation: Derive inverse trig derivatives by implicit differentiation of $\sin y = x$, then $\cos y = \sqrt{1 - x^2}$.

AP Calc BC Tip: *you never need to derive them on the exam -- memorize the three main ones.*

Example: $y = \arcsin x \Rightarrow \sin y = x \Rightarrow \cos y \cdot y' = 1 \Rightarrow y' = 1/\sqrt{1 - x^2}$.

53 Linear Approximation at a Point

Explanation: The tangent line $L(x) = f(a) + f'(a)(x - a)$ approximates f near a . The error is roughly $(1/2) f''(c)(x - a)^2$.

AP Calc BC Tip: *linear approximation is the $n = 1$ Taylor polynomial -- Unit 10 generalizes this.*

Example: $\sqrt{4.02} \sim 2 + (1/4)(0.02) = 2.005$.

Explanation: $dy = f'(x) dx$ approximates the change in y for a small change dx ; use it for error propagation estimates.

AP Calc BC Tip: $relative\ error = dy/y$; $percent\ error = 100 * |dy/y|$.

Example: If $V = s^3$ with $s = 10$ and $ds = 0.1$, $dV = 3(100)(0.1) = 30$ cubic units.

Unit 4: Unit 4: Contextual Applications of Differentiation

~6% of the exam. Interpreting derivatives in real-world and motion contexts.

55 Velocity & Acceleration

Explanation: For position $s(t)$: velocity $v(t) = s'(t)$, acceleration $a(t) = v'(t) = s''(t)$. Speed = $|v(t)|$.

AP Calc BC Tip: particle moving right when $v > 0$, left when $v < 0$; speeding up when v and a have the SAME sign.

Example: $s(t) = t^3 - 6t^2$; $v(t) = 3t^2 - 12t$; $a(t) = 6t - 12$.

56 Position, Velocity & Displacement

Explanation: Displacement over $[a, b] = s(b) - s(a) = \text{integral of } v(t)$. Total distance traveled = integral of $|v(t)|$.

AP Calc BC Tip: displacement vs distance: distance ignores direction (integrate absolute velocity).

Example: $v(t) = t - 2$ on $[0, 4]$: displacement = integral $(t-2) dt = 0$; distance = 4.

57 Speeding Up & Slowing Down

Explanation: A particle speeds up when velocity and acceleration have the same sign; slows down when they differ.

AP Calc BC Tip: check the signs of $v(t)$ and $a(t)$ -- same sign means speeding up.

Example: $v = -3$, $a = -2$: both negative, so speeding up (moving left, accelerating left).

58 Marginal Analysis

Explanation: Marginal cost $C'(x)$ approximates the cost of producing one more unit; marginal revenue $R'(x)$; profit maximized where $MR = MC$.

AP Calc BC Tip: profit max: $R'(x) = C'(x)$; check that it is a max (second derivative or sign change).

Example: If $C(x) = x^2 + 10$, then $C'(x) = 2x$ is the marginal cost.

59 Rates of Change in Context

Explanation: Interpret $f'(a)$ as 'the rate at which f changes per unit of x at $x = a$ ' with proper units.

AP Calc BC Tip: the interpretation point requires UNITS: 'thousands of dollars per year', etc.

Example: If $P'(5) = 300$, the population grows at 300 people per year at year 5.

60 Related Rates: Cones & Tanks

Explanation: For filling-tank problems, express volume in one variable using similar triangles, then differentiate in t .

AP Calc BC Tip: similar triangles give r/h constant -- substitute before differentiating to avoid product rule pain.

Example: Cone of radius 3, height 6: $r = h/2$, so $V = (1/3)\pi (h/2)^2 h = (\pi/12) h^3$.

61 Related Rates: Ladders & Shadows

Explanation: Ladder: $x^2 + y^2 = L^2$. Shadow: similar triangles link the person's height to the shadow length.

AP Calc BC Tip: shadow problems: dx/dt (shadow tip) = rate the shadow grows; the person's speed is given.

Example: Ladder of length 13: $2x dx/dt + 2y dy/dt = 0$ with $x^2 + y^2 = 169$.

62 Related Rates: Circles & Spheres

Explanation: Circle area $A = \pi r^2$; sphere volume $V = (4/3)\pi r^3$. $dA/dt = 2\pi r dr/dt$; $dV/dt = 4\pi r^2 dr/dt$.

AP Calc BC Tip: these are the cleanest related-rate formulas -- memorize them.

Example: If $dr/dt = 2$ and $r = 3$, $dA/dt = 2\pi (3)(2) = 12\pi$.

63 The Mean Value Theorem in Context

Explanation: MVT guarantees that average rate of change equals instantaneous rate at some interior point.

AP Calc BC Tip: *phrase the conclusion: 'there exists c in (a,b) such that $f'(c)$ equals the average rate.'*

Example: If $f(0) = 1$ and $f(4) = 9$, then $f'(c) = 2$ for some c in $(0, 4)$.

64 Interpreting the Derivative Graph

Explanation: Given the graph of f' , read f 's increases/decreases (sign of f'), extrema ($f' = 0$ with sign change), and concavity (f'' sign via f' slope).

AP Calc BC Tip: *f' crosses zero from $+$ to $-$ $\Rightarrow$ local max; from $-$ to $+$ $\Rightarrow$ local min.*

Example: If $f' > 0$ then $f' < 0$ at $x = 2$, f has a local maximum at $x = 2$.

65 The Second Derivative Test

Explanation: If $f'(c) = 0$ and $f''(c) > 0$, then c is a local minimum; if $f''(c) < 0$, a local maximum; if $f''(c) = 0$, inconclusive.

AP Calc BC Tip: *the second derivative test only applies at critical points where $f'(c) = 0$.*

Example: $f(x) = x^4 - 4x^2$: at $x = 0$, $f'' = -8 < 0$ (local max); at $x = \pm\sqrt{2}$, $f'' > 0$ (min).

66 Concavity & Points of Inflection

Explanation: f is concave up where $f'' > 0$, concave down where $f'' < 0$. An inflection point is where concavity CHANGES (f'' crosses zero).

AP Calc BC Tip: *$f'' = 0$ alone is not an inflection point -- the sign must change.*

Example: $f(x) = x^3$ has an inflection at 0: $f'' = 6x$ changes sign there.

67 Optimization: Setup

Explanation: Optimization: identify the quantity to maximize/minimize, write it as a function of one variable, find critical points, check endpoints.

AP Calc BC Tip: *name the constraint equation and the objective; substitute to get one variable.*

Example: Maximize area $A = xy$ with fence $2x + y = 100$: $A = x(100 - 2x)$.

68 Optimization: Solve

Explanation: Find candidates at critical points AND endpoints; compare values; answer in the problem's units.

AP Calc BC Tip: *justify with the first or second derivative test, not just by inspection.*

Example: $A = x(100 - 2x)$ peaks at $x = 25$ ($A'' = -4 < 0$), giving $A = 1250$.

69 The Candidate Test

Explanation: On a closed interval, the extrema of a continuous function occur at critical points or endpoints -- evaluate all candidates.

AP Calc BC Tip: *never skip endpoints on closed-interval optimization.*

Example: $f(x) = x^3 - 3x$ on $[0, 2]$: candidates $x = 0, 1, 2$ with $f = 0, -2, 2 \rightarrow$ max 2, min -2.

70 Rates of Change of the Derivative

Explanation: f'' measures how fast the slope changes: positive f'' means slopes are increasing (concave up).

AP Calc BC Tip: *interpret f'' as 'the rate of change of the rate of change.'*

Example: If $f'' > 0$ everywhere, f 's graph opens upward and slopes rise.

71 Average Rate of Change of a Function

Explanation: ARC on $[a, b] = (f(b) - f(a))/(b - a)$; it is the slope of the secant line and equals the average value of f' by MVT.

AP Calc BC Tip: *ARC = slope of secant; IROC = slope of tangent.*

Example: $f(x) = x^2$ on $[1, 3]$: ARC = $(9 - 1)/2 = 4$.

Explanation: For vertical motion (rockets, falling objects), up is positive: $v > 0$ rising, $a = -32 \text{ ft/s}^2$ (or -9.8 m/s^2) near Earth.

AP Calc BC Tip: at maximum height, $v = 0$ -- that is the critical point of $s(t)$.

Example: $s(t) = -16t^2 + 64t$: max height where $v = -32t + 64 = 0$, $t = 2$, $s(2) = 64 \text{ ft}$.

Unit 5: Unit 5: Analytical Applications of Differentiation

~8% of the exam. Using derivatives to analyze function behavior and graphs.

73 The First Derivative Test

Explanation: If f' changes + to - at a critical point c , f has a local max at c ; if - to +, a local min.

AP Calc BC Tip: use the sign chart of f' around critical points.

Example: $f = (x - 2)(x + 1)$: sign + - + gives max at $x = -1$ and min at $x = 2$.

74 The Second Derivative Test

Explanation: At a critical point c with $f'(c) = 0$: $f''(c) > 0$ gives a local min; $f''(c) < 0$ gives a local max; $f''(c) = 0$ is inconclusive.

AP Calc BC Tip: when $f''(c) = 0$, fall back to the first derivative test.

Example: $f(x) = x^4$: $f'(0) = 0$ and $f''(0) = 0$, so the test is inconclusive -- f actually has a min at 0.

75 Global vs Local Extrema

Explanation: A global (absolute) extremum is the largest/smallest value on the whole domain or interval; local extrema are peaks/valleys in neighborhoods.

AP Calc BC Tip: on closed intervals compare critical values AND endpoint values for the global answer.

Example: $f(x) = x^2$ on $[-2, 1]$: global min 0 at $x = 0$, global max 4 at $x = -2$.

76 Concavity Testing

Explanation: $f'' > 0 \Rightarrow$ concave up (holds water); $f'' < 0 \Rightarrow$ concave down. Sign changes of f'' mark inflection points.

AP Calc BC Tip: concavity is a f'' question -- read the graph of f'' directly when given.

Example: $f(x) = -x^4$ is concave down everywhere ($f'' = -12x^2 < 0$ except at 0).

77 Inflection Points

Explanation: An inflection point is a point where the graph changes concavity; $f'' = 0$ or undefined there, and f'' must change sign.

AP Calc BC Tip: verify sign change of f'' ; a zero of f'' without a sign change is not an inflection point.

Example: $f(x) = x^3$: $f'' = 6x$ changes sign at 0, so $(0, 0)$ is an inflection point.

78 Curve Sketching Summary

Explanation: Sketch: domain, intercepts, asymptotes, critical points, extrema, concavity, inflection points, end behavior.

AP Calc BC Tip: on FRQs, each labeled feature earns a point -- include the reasoning, not just the graph.

Example: A rational function's sketch needs vertical/horizontal asymptotes plus the sign chart.

79 The Intermediate Value Theorem for Derivatives

Explanation: Derivatives have the intermediate value property (Darboux's theorem): f' takes all values between $f'(a)$ and $f'(b)$.

AP Calc BC Tip: this underlies why sign changes of f' force critical points between them.

Example: If $f'(0) = -1$ and $f'(2) = 3$, then $f' = 0$ somewhere in $(0, 2)$.

80 L'Hopital in Unit 5 Context

Explanation: L'Hopital's rule resolves $0/0$ and ∞/∞ limits; it is essential for evaluating indeterminate forms in BC.

AP Calc BC Tip: always check the indeterminate form before applying; otherwise the result is invalid.

Example: $\lim (\sin x)/x$ as $x \rightarrow 0 = \lim (\cos x)/1 = 1$.

81 Behavior of f Given f' and f''

Explanation: Given graphs of f' and f'' , reconstruct f : f increasing where $f' > 0$, concave up where $f'' > 0$, extrema where $f' = 0$ with sign change.

AP Calc BC Tip: read f' as the slope of f -- the graph of f' is the derivative graph of f .

Example: Where $f' > 0$, the graph of f is increasing.

82 The Relationship Between f , f' , and f''

Explanation: Each derivative strips one layer of behavior: f 's heights, f 's slopes, f 's curvature.

AP Calc BC Tip: on graph-matching questions, match zeros and signs across the three graphs.

Example: If f has a local max, $f' = 0$ there and $f'' \leq 0$ (if it exists).

83 Justifying Extrema with Derivatives

Explanation: To justify a local max: show f' changes from positive to negative at c (first derivative test).

AP Calc BC Tip: write the justification as a mini-argument: sign of f' on each side.

Example: Since $f' > 0$ on $(0, c)$ and $f' < 0$ on (c, d) , f has a local maximum at c .

84 Critical Points & Where f' Does Not Exist

Explanation: Critical points: $f'(c) = 0$ OR $f'(c)$ does not exist (c in f 's domain). Extrema can occur at either type.

AP Calc BC Tip: check corners and cusps as candidates, not just zeros of f' .

Example: $f(x) = |x|$ has a critical point at 0 where f' does not exist -- and a global min there.

85 Optimization with Constraints

Explanation: Use the constraint to reduce the objective to one variable; a common BC trap is optimizing with two variables left.

AP Calc BC Tip: write the constraint FIRST, then substitute.

Example: Maximize xy subject to $x + y = 10$: $y = 10 - x$, so maximize $x(10 - x)$.

86 The Closed Interval Method

Explanation: To find absolute extrema on $[a, b]$: evaluate f at all critical points in (a, b) and at a and b ; pick the largest/smallest.

AP Calc BC Tip: this is the guaranteed-complete method -- list every candidate value.

Example: $f(x) = x^3 - 3x^2$ on $[0, 3]$: $f(0) = 0$, $f(1) = -2$, $f(2) = -4$, $f(3) = 0 \rightarrow$ max 0, min -4.

87 Rates of Change of Rates

Explanation: Acceleration is the rate of change of velocity; jerk (rarely tested) is the rate of change of acceleration.

AP Calc BC Tip: higher derivatives in motion: s , $v = s'$, $a = s''$.

Example: $s(t) = 2t^3$: $v = 6t^2$, $a = 12t$.

88 Mean Value Theorem Applications

Explanation: MVT implies f is constant iff $f' = 0$ on an interval; f is increasing iff $f' \geq 0$.

AP Calc BC Tip: use $f' \geq 0$ to prove monotonicity on an interval.

Example: If $f'(x) = 3$ for all x , then $f(x) = 3x + C$.

89 Finding Where f is Increasing/Decreasing

Explanation: f is increasing on intervals where $f' > 0$ and decreasing where $f' < 0$; endpoints excluded.

AP Calc BC Tip: state intervals in x -values, and use open intervals unless the function is defined at endpoints.

Example: If $f'(x) = (x - 1)(x - 3)$, f increases on $(-\infty, 1) \cup (3, \infty)$.

Explanation: For parametric curves, compute $d^2y/dx^2 = d/dt(dy/dx) / (dx/dt)$ to test concavity.

AP Calc BC Tip: *the parametric second derivative is BC material -- never divide the second derivatives directly.*

Example: $x = t^2, y = t^3$: $d^2y/dx^2 = 3/(4t)$; for $t < 0$ the curve is concave down.

Unit 6: Unit 6: Integration & Accumulation of Change

~17-18% of the exam. The heaviest AB unit, plus BC techniques: parts, partial fractions, improper integrals.

91 The Fundamental Theorem of Calculus Part 1

Explanation: If $F(x) = \text{integral from } a \text{ to } x \text{ of } f(t) \, dt$, then $F'(x) = f(x)$. Differentiation undoes definite integration with variable upper limit.

AP Calc BC Tip: the FTC part 1 chain rule form: $d/dx \text{ integral from } a \text{ to } g(x) = f(g(x)) g'(x)$.

Example: $d/dx \text{ integral from } 0 \text{ to } x^2 \text{ of } \sin(t) \, dt = \sin(x^2) \cdot 2x$.

92 The Fundamental Theorem of Calculus Part 2

Explanation: $\text{integral from } a \text{ to } b \text{ of } f(x) \, dx = F(b) - F(a)$ where F is any antiderivative of f .

AP Calc BC Tip: FTC part 2 evaluates definite integrals -- find the antiderivative, then subtract.

Example: $\text{integral from } 1 \text{ to } 2 \text{ of } x^2 \, dx = 8/3 - 1/3 = 7/3$.

93 Accumulation Functions

Explanation: $F(x) = \text{integral from } a \text{ to } x \text{ of } f(t) \, dt$ accumulates the signed area under f from a to x .

AP Calc BC Tip: $F(a) = 0$; F is increasing where $f > 0$; F 's extrema where $f = 0$ with sign change.

Example: $F(x) = \text{integral from } 0 \text{ to } x \text{ of } t \, dt = x^2/2$, so $F(3) = 9/2$.

94 Riemann Sums

Explanation: Approximate integrals by rectangles (left, right, midpoint) or trapezoids; more subdivisions = better approximation.

AP Calc BC Tip: left sums overestimate increasing functions, underestimate decreasing ones; right sums reverse.

Example: Left sum of $f(x) = x$ on $[0, 2]$ with 2 rectangles: $0 + 1 = 1$ (underestimate).

95 The Trapezoidal Rule

Explanation: Approximate integral $\sim (b - a)/(2n) [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$.

AP Calc BC Tip: trapezoid rule = average of left and right rectangle rules.

Example: With 4 trapezoids on $[0, 2]$, weight interior heights by 2.

96 The Power Rule for Integrals

Explanation: $\text{integral } x^n \, dx = x^{(n+1)}/(n+1) + C$ for $n \neq -1$; $\text{integral } x^{-1} \, dx = \ln|x| + C$.

AP Calc BC Tip: $n = -1$ is the ONE exception -- it integrates to a logarithm, not a power.

Example: $\text{integral } x^{-2} \, dx = -x^{-1} + C$; $\text{integral } 1/x \, dx = \ln|x| + C$.

97 U-Substitution

Explanation: Reverse the chain rule: choose $u = \text{inner function}$, $du = u' \, dx$, rewrite the integral entirely in u , integrate, substitute back.

AP Calc BC Tip: after substituting, the integral must contain NO x -- every x must be replaced by u or du .

Example: $\text{integral } 2x \cos(x^2) \, dx$ with $u = x^2$: $\text{integral } \cos u \, du = \sin(x^2) + C$.

98 Integration by Parts

Explanation: $\text{integral } u \, dv = uv - \text{integral } v \, du$. Choose u by LIATE (Logs, Inverse trig, Algebraic, Trig, Exponential) order.

AP Calc BC Tip: BC-only core technique -- pick u to simplify upon differentiation, dv to integrate cleanly.

Example: $\text{integral } x e^x \, dx$: $u = x$, $dv = e^x \, dx \rightarrow x e^x - \text{integral } e^x \, dx = x e^x - e^x + C$.

99 Integration by Parts: Repeated

Explanation: When the first parts application leaves another product, apply parts again (or tabular integration).

AP Calc BC Tip: *tabular integration handles $x^n e^x$ and $x^n \sin x$ quickly.*

Example: integral $x^2 e^x dx$ needs parts twice: $x^2 e^x - 2x e^x + 2 e^x + C$.

100 Integration by Parts: The Cyclic Case

Explanation: For integral $e^x \sin x dx$, parts twice returns the original integral -- solve the resulting equation for it.

AP Calc BC Tip: *label the original integral I and solve $I = \dots \pm I$.*

Example: $I = \text{integral } e^x \sin x dx$: parts twice gives $I = e^x(\sin x - \cos x)/2 + C$.

101 Partial Fractions

Explanation: Decompose a rational function into a sum of simpler fractions: distinct linear factors $A/(x-a) + B/(x-b)$, repeated factors, irreducible quadratics.

AP Calc BC Tip: *BC-only technique -- factor the denominator fully first, then solve for the coefficients.*

Example: integral $1/(x^2 - 1) dx = \text{integral } [1/2/(x-1) - 1/2/(x+1)] dx = (1/2) \ln|(x-1)/(x+1)| + C$.

102 Partial Fractions: Repeated & Quadratic Factors

Explanation: Repeated factor $(x-a)^n$ needs terms $A_1/(x-a) + \dots + A_n/(x-a)^n$; irreducible $x^2 + bx + c$ needs $(Ax + B)/(x^2 + bx + c)$.

AP Calc BC Tip: *for quadratic denominators that don't factor, complete the square and use arctan.*

Example: integral $1/(x^2 + 1) dx = \arctan x + C$; integral $1/(x^2 + 4) dx = (1/2) \arctan(x/2) + C$.

103 Improper Integrals

Explanation: Integrals with infinite limits or infinite discontinuities: replace with a limit, evaluate, and check convergence.

AP Calc BC Tip: *an improper integral converges if the limiting value is finite; diverges to inf or does not exist otherwise.*

Example: integral from 1 to inf of $1/x^2 dx = \lim [-1/b + 1] = 1$ (converges).

104 Improper Integrals: p-Integrals

Explanation: integral from 1 to inf of $1/x^p dx$ converges iff $p > 1$; integral from 0 to 1 of $1/x^p$ converges iff $p < 1$.

AP Calc BC Tip: *p-integral convergence is the benchmark for comparison tests later.*

Example: integral from 1 to inf of $1/\sqrt{x} dx$ diverges because $p = 1/2 < 1$.

105 Integrals of Trig Functions

Explanation: integral $\sin x dx = -\cos x + C$; integral $\cos x dx = \sin x + C$; integral $\sec^2 x dx = \tan x + C$; integral $\sec x \tan x dx = \sec x + C$.

AP Calc BC Tip: *the minus sign: sin integrates to -cos.*

Example: integral $3 \cos(2x) dx = (3/2) \sin(2x) + C$.

106 Trigonometric Integrals (BC)

Explanation: Use identities: $\sin^2 x = (1 - \cos 2x)/2$, $\cos^2 x = (1 + \cos 2x)/2$, $\sin x \cos x = (1/2) \sin 2x$; powers via reduction.

AP Calc BC Tip: *even powers of sin/cos use half-angle identities; odd powers save one factor for u-sub.*

Example: integral $\sin^2 x dx = x/2 - \sin(2x)/4 + C$.

107 The Average Value of a Function

Explanation: Average value of f on $[a, b] = (1/(b-a)) \text{ integral from } a \text{ to } b \text{ of } f(x) dx$.

AP Calc BC Tip: *average value = total area divided by interval length -- the FTC/MVT bridge.*

Example: Average of $f(x) = x$ on $[0, 2]$ is $(1/2)(2) = 1$.

Explanation: Area between f and g on $[a, b]$ = integral $|f - g| dx$; integrate top minus bottom (or right minus left for y -functions).

AP Calc BC Tip: *find intersection points for the interval; if curves cross, split the integral.*

Example: Area between $y = x^2$ and $y = x$ on $[0, 1]$ = integral $(x - x^2) dx = 1/6$.

Unit 7: Unit 7: Differential Equations

~6-7% of the exam. Separable equations, slope fields, and Euler's method (BC).

109 Separable Differential Equations

Explanation: A differential equation $dy/dx = f(x)g(y)$ separates: put all y's with dy and all x's with dx, integrate both sides.

AP Calc BC Tip: separate *FIRST*, integrate both sides, then solve for y explicitly when possible.

Example: $dy/dx = y$: $dy/y = dx$, $\ln|y| = x + C$, $y = Ce^x$.

110 General vs Particular Solutions

Explanation: The general solution has an arbitrary constant C; a particular solution uses an initial condition to fix C.

AP Calc BC Tip: plug the initial condition into the general solution to solve for C.

Example: $y = Ce^x$ with $y(0) = 3$ gives $C = 3$, so $y = 3e^x$.

111 Slope Fields

Explanation: A slope field draws short line segments of slope dy/dx at grid points, visualizing solution curves.

AP Calc BC Tip: match a differential equation to its slope field by testing a few points' slopes.

Example: $dy/dx = x$ gives slopes increasing with x; $dy/dx = y$ gives slopes growing with height.

112 Matching Slope Fields

Explanation: Check the sign of dy/dx in each quadrant and along axes to identify the correct field.

AP Calc BC Tip: $dy/dx = x$ has zero slope along the y-axis; $dy/dx = y$ has zero slope along the x-axis.

Example: For $dy/dx = x - y$, slopes are zero on the line $y = x$.

113 Euler's Method

Explanation: Approximate a solution: $y(n+1) = y(n) + f(x(n), y(n)) \cdot h$ with step $h = (b - a)/n$.

AP Calc BC Tip: BC-specific: each step uses the *CURRENT* (x, y) to get the slope; recompute f each step.

Example: $dy/dx = x + y$, $y(0) = 1$, $h = 0.5$: $y(0.5) = 1 + (0 + 1)(0.5) = 1.5$; $y(1) = 1.5 + (0.5 + 1.5)(0.5) = 2.5$.

114 Euler's Method Error

Explanation: Euler's method accumulates error; smaller step sizes give better approximations but more steps.

AP Calc BC Tip: the approximation *UNDERESTIMATES* convex solutions (concave up) and overestimates concave-down ones.

Example: For $y' = y$, Euler underestimates e^x because the solution is convex.

115 Exponential Growth & Decay

Explanation: $dy/dt = ky$ solves to $y = y_0 e^{(kt)}$; $k > 0$ growth, $k < 0$ decay. Half-life: $t = \ln(2)/|k|$.

AP Calc BC Tip: identify the model from wording: 'grows proportionally to its size' means $dy/dt = ky$.

Example: A population doubling every 10 years has $y = y_0 2^{(t/10)}$.

116 The Logistic Model

Explanation: $dP/dt = kP(1 - P/M)$: growth limited by carrying capacity M. Solutions are S-curves with inflection at $P = M/2$.

AP Calc BC Tip: logistic growth: P approaches M; fastest growth at half the carrying capacity.

Example: $dP/dt = 0.1P(1 - P/1000)$: carrying capacity 1000, inflection at $P = 500$.

117 Newton's Law of Cooling

Explanation: $dT/dt = k(T - T_{\text{env}})$: the object's temperature changes proportionally to the difference from the environment.

AP Calc BC Tip: solutions $T = T_{\text{env}} + (T_0 - T_{\text{env}}) e^{k(t)}$; k is negative.

Example: A 200-degree object in a 70-degree room cools toward 70 exponentially.

118 Drawing Solution Curves on Slope Fields

Explanation: Solution curves follow the field's arrows, threading through the segments smoothly.

AP Calc BC Tip: sketch curves that respect the field everywhere; use the given initial point.

Example: Through (0, 0) with $dy/dx = x$, the curve is $y = x^2/2$.

119 Solving $dy/dx = f(x) g(y)$ with Initial Values

Explanation: Use definite integrals: $\int_{y_0}^y 1/g(u) du = \int_{x_0}^x f(t) dt$.

AP Calc BC Tip: the definite-integral form avoids solving for C entirely.

Example: $dy/dx = 2x/y$ with $y(0) = 1$: $y^2/2 - 1/2 = x^2$, so $y = \sqrt{2x^2 + 1}$.

120 Verifying Solutions

Explanation: Plug a proposed solution (and its derivative) into the differential equation; both sides must match.

AP Calc BC Tip: differentiate the proposed y , substitute, and simplify to confirm.

Example: $y = e^{2x}$ solves $y' = 2y$ since $(e^{2x})' = 2e^{2x}$.

121 The Constant of Integration from Initial Conditions

Explanation: Always include $+ C$ after integrating; the initial condition determines its value uniquely.

AP Calc BC Tip: forgetting $+ C$ turns a family of solutions into a single wrong answer.

Example: $dy/dx = 3x^2$, $y(1) = 5$: $y = x^3 + C$, and $5 = 1 + C$ gives $C = 4$.

122 Applications: Mixing & Temperature

Explanation: Mixing problems set up rate of change = rate in - rate out; cooling uses the difference model.

AP Calc BC Tip: write the differential equation from the word problem, then solve with separation.

Example: Salt in a tank: $dS/dt = (\text{concentration in})(\text{flow in}) - (S/V)(\text{flow out})$.

123 Logistic Differential Equation Solutions

Explanation: The logistic solution $P(t) = M / (1 + A e^{-kt})$ with $A = (M - P_0)/P_0$.

AP Calc BC Tip: you may be asked to derive this from the separable form using partial fractions.

Example: $P_0 = 100$, $M = 1000$, $k = 0.1$: $A = 9$, so $P(t) = 1000/(1 + 9e^{-0.1t})$.

124 Equilibrium Solutions

Explanation: Constant solutions where $dy/dx = 0$ are equilibria; stable if nearby solutions approach, unstable if they flee.

AP Calc BC Tip: in the logistic model, $P = 0$ is unstable, $P = M$ is stable.

Example: $dy/dt = y(2 - y)$: equilibria $y = 0$ (unstable) and $y = 2$ (stable).

125 Motion with Proportional Drag

Explanation: $dv/dt = g - kv$ models a falling object with air resistance; terminal velocity = g/k .

AP Calc BC Tip: terminal velocity is the equilibrium of the velocity equation.

Example: $dv/dt = 32 - 4v$: terminal velocity is 8 ft/s.

Explanation: Curves that cross a given family at right angles: swap dy/dx for $-dx/dy$ in the family's equation.

AP Calc BC Tip: *mostly enrichment -- know the concept, rarely tested.*

Example: For circles $x^2 + y^2 = C$, orthogonal trajectories are lines $y = mx$.

Unit 8: Unit 8: Applications of Integration

~6-7% of the exam. Area, volume, arc length, and accumulated change.

127 Area Between Curves

Explanation: Area = integral from a to b of $[f(x) - g(x)] dx$ with f above g; find bounds at intersections.

AP Calc BC Tip: draw the region; if unsure which curve is on top, integrate $|f - g|$.

Example: Area between $y = x$ and $y = x^3$ on $[0, 1] = \int_0^1 (x - x^3) dx = 1/4$.

128 Volume by Discs

Explanation: Rotate about the x-axis: $V = \pi \int [R(x)]^2 dx$, where R is the distance from the axis to the curve.

AP Calc BC Tip: discs for rotation about the axis of the region; R is the outer radius.

Example: $V = \pi \int_0^1 (x)^2 dx = \pi/3$ for $y = x$ about the x-axis.

129 Volume by Washers

Explanation: Rotate about an axis outside the region: $V = \pi \int [R^2 - r^2] dx$ (outer radius squared minus inner radius squared).

AP Calc BC Tip: identify the hole: the inner radius is the axis-to-inner-curve distance.

Example: Rotating $y = x$ and $y = x^2$ about the x-axis: washers with $R = x$, $r = x^2$.

130 Volume by Cylindrical Shells

Explanation: $V = 2\pi \int r(x) h(x) dx$: shells integrate the surface area of cylinders (radius times height).

AP Calc BC Tip: shells are best when rotating about the y-axis with x-integration, or vice versa.

Example: $V = 2\pi \int_0^1 x(x - x^2) dx$ for $y = x$, $y = x^2$ about the y-axis.

131 Choosing Discs vs Shells

Explanation: Discs/washers integrate perpendicular to the axis; shells integrate parallel to the axis. Pick the simpler integrand.

AP Calc BC Tip: if the axis is a boundary of the region, discs usually work; if not, try shells.

Example: Rotating about $x = 2$ with functions of x: shells (radius = $2 - x$) are often cleaner.

132 Volume by Cross Sections

Explanation: $V = \int A(x) dx$ where $A(x)$ is the area of a known cross section (squares, semicircles, equilateral triangles).

AP Calc BC Tip: the cross section's size depends on the region's vertical or horizontal width.

Example: Square cross sections perpendicular to x-axis: $A(x) = [f(x) - g(x)]^2$.

133 Average Value of a Function

Explanation: Average value = $(1/(b - a)) \int_a^b f(x) dx$. Interpret as the height of a rectangle with equal area.

AP Calc BC Tip: connect to MVT for integrals: the average is attained at some point.

Example: Average of $\sin x$ on $[0, \pi] = (1/\pi)(2) = 2/\pi$.

134 The Mean Value Theorem for Integrals

Explanation: If f is continuous on $[a, b]$, there exists c in (a, b) with $f(c) = (1/(b - a)) \int_a^b f(x) dx$.

AP Calc BC Tip: 'the average value is attained' -- a justification phrase.

Example: For $f(x) = x$ on $[0, 2]$, average 1 is attained at $c = 1$.

135 Accumulated Change from a Rate

Explanation: Net change = integral from a to b of rate(t) dt; total change of F equals the accumulated rate.

AP Calc BC Tip: the FTC: $F(b) = F(a) + \text{integral of } F'$.

Example: If water flows in at $r(t)$, the total added by time T is integral from 0 to T of $r(t)$ dt.

136 Displacement vs Distance (Revisited)

Explanation: Displacement = integral $v(t)$ dt; distance = integral $|v(t)|$ dt.

AP Calc BC Tip: split at zeros of $v(t)$ to compute total distance.

Example: $v(t) = t - 3$ on $[0, 5]$: distance = integral $|t - 3|$ dt = $9/2 + 2 = 13/2$.

137 Arc Length (BC)

Explanation: Length of $y = f(x)$ on $[a, b]$: $L = \text{integral} \sqrt{1 + (f'(x))^2} dx$.

AP Calc BC Tip: BC-specific; also parametric form $L = \text{integral} \sqrt{(dx/dt)^2 + (dy/dt)^2} dt$.

Example: Arc length of $y = x^{3/2}$ on $[0, 4]$: $L = \text{integral} \sqrt{1 + (9/4)x} dx$.

138 Parametric Arc Length (BC)

Explanation: For $x(t), y(t)$ on $[a, b]$: $L = \text{integral from } a \text{ to } b \text{ of } \sqrt{(dx/dt)^2 + (dy/dt)^2} dt$.

AP Calc BC Tip: this is the speed of the particle integrated over time -- total distance traveled.

Example: $x = \cos t, y = \sin t$ on $[0, 2\pi]$: $L = \text{integral } 1 dt = 2\pi$.

139 Surface Area of Revolution (BC)

Explanation: $S = 2\pi \text{ integral } y \sqrt{1 + (y')^2} dx$ for rotation about the x-axis.

AP Calc BC Tip: BC-only; combine the arc-length element with the circumference $2\pi r$.

Example: Surface area of a sphere: $S = 2\pi \text{ integral from } -R \text{ to } R \text{ of } \sqrt{R^2 - x^2} * R/\sqrt{R^2 - x^2} dx = 4\pi R^2$.

140 Area in Polar Coordinates (BC)

Explanation: Area swept by $r = f(\theta)$ from a to b: $A = (1/2) \text{ integral } r^2 d\theta$.

AP Calc BC Tip: BC-specific: $(1/2) r^2 d(\theta)$, NOT integral $r d(\theta)$.

Example: Area of one petal of $r = \sin(2\theta)$ = $(1/2) \text{ integral from } 0 \text{ to } \pi/2 \text{ of } \sin^2(2\theta) d\theta = \pi/8$.

141 Arc Length in Polar (BC)

Explanation: $L = \text{integral} \sqrt{r^2 + (dr/d\theta)^2} d\theta$ for polar curves.

AP Calc BC Tip: analogous to parametric length with $x = r \cos \theta, y = r \sin \theta$.

Example: Circumference of $r = a$: $L = \text{integral from } 0 \text{ to } 2\pi \text{ of } \sqrt{a^2 + 0} d\theta = 2\pi a$.

142 Work (BC Plus)

Explanation: Work = integral $F(x) dx$: force times distance, integrated when the force varies.

AP Calc BC Tip: spring work uses Hooke's law $F = kx$; pumping water uses weight density times volume.

Example: Spring with $k = 5$ stretched 3 ft: $W = \text{integral from } 0 \text{ to } 3 \text{ of } 5x dx = 45/2 \text{ ft}\cdot\text{lb}$.

143 Modeling with Accumulation

Explanation: Set up integrals from word problems: identify the rate being accumulated and the window of time.

AP Calc BC Tip: write the integral FIRST, then evaluate; units confirm the setup.

Example: Oil leaking at $r(t)$ barrels/hour: total leak over $[0, 6]$ = integral from 0 to 6 of $r(t)$ dt barrels.

Explanation: The integral of a rate of change equals the net change of the quantity: $\int F'(t) dt = F(b) - F(a)$.

AP Calc BC Tip: *this is FTC part 2 restated -- the workhorse of context problems.*

Example: Population $P'(t) = 50t$: net change from $t = 1$ to $3 = \int 50t dt = 400$.

Unit 9: Unit 9: Parametric Equations, Polar Coordinates & Vector-Valued Functions

~10-11% of the exam. BC-only unit: motion in the plane.

145 Parametric Equations

Explanation: $x = f(t)$, $y = g(t)$ trace a curve as t varies; t is the parameter (often time).

AP Calc BC Tip: *eliminate t to find the Cartesian equation, or work directly in t .*

Example: $x = \cos t$, $y = \sin t$ gives the unit circle $x^2 + y^2 = 1$.

146 Parametric Derivatives

Explanation: $dy/dx = (dy/dt)/(dx/dt)$ when $dx/dt \neq 0$; slope of the tangent in parametric form.

AP Calc BC Tip: *do NOT use dy/dt alone -- divide by dx/dt .*

Example: $x = t^2$, $y = t^3$: $dy/dx = (3t^2)/(2t) = 3t/2$.

147 Parametric Second Derivative

Explanation: $d^2y/dx^2 = d/dt(dy/dx) / (dx/dt)$. Differentiate dy/dx with respect to t , then divide by dx/dt .

AP Calc BC Tip: *this is the #1 BC parametric trap -- never divide d^2y/dt^2 by d^2x/dt^2 .*

Example: $x = t^2$, $y = t^3$: $d^2y/dx^2 = (3t)/(2t) = 3/(4t)$.

148 Horizontal & Vertical Tangents (Parametric)

Explanation: Horizontal tangents where $dy/dt = 0$ (and $dx/dt \neq 0$); vertical tangents where $dx/dt = 0$ (and $dy/dt \neq 0$).

AP Calc BC Tip: *check the OTHER derivative is nonzero to avoid cusps.*

Example: $x = t^2$, $y = t^3 - 3t$: $dy/dt = 3(t^2 - 1) = 0$ at $t = \pm 1$ (horizontal tangents).

149 Speed in Parametric Motion

Explanation: Speed = $\sqrt{(dx/dt)^2 + (dy/dt)^2} = |v(t)|$, the magnitude of the velocity vector.

AP Calc BC Tip: *speed is NOT $|dy/dx|$ -- it is the magnitude of the velocity vector.*

Example: $x = 3 \cos t$, $y = 3 \sin t$: speed = 3 everywhere (uniform circular motion).

150 Acceleration Vectors (Parametric)

Explanation: Acceleration = $(d^2x/dt^2, d^2y/dt^2)$; its magnitude is $\sqrt{(x'')^2 + (y'')^2}$.

AP Calc BC Tip: *acceleration is a vector; its magnitude is the second-derivative speed.*

Example: $x = t^2$, $y = t^3$: $a = (2, 6t)$.

151 Vector-Valued Functions

Explanation: Position $r(t) = \langle x(t), y(t) \rangle$; velocity $v(t) = r'(t)$; acceleration $a(t) = v'(t) = r''(t)$.

AP Calc BC Tip: *differentiate component-wise; the vector notation is BC-standard.*

Example: $r(t) = \langle t^2, \sin t \rangle$: $v = \langle 2t, \cos t \rangle$, $a = \langle 2, -\sin t \rangle$.

152 Motion Along a Parametric Curve

Explanation: The particle moves up/right when the relevant velocity component is positive; combine signs for direction.

AP Calc BC Tip: *direction = sign of dx/dt (horizontal) and dy/dt (vertical).*

Example: $dx/dt > 0$ and $dy/dt < 0$: the particle moves right and down.

153 Polar Coordinates Basics

Explanation: Polar: (r, θ) with $x = r \cos \theta$, $y = r \sin \theta$; $r^2 = x^2 + y^2$, $\tan \theta = y/x$.

AP Calc BC Tip: convert with $x = r \cos \theta$ and $y = r \sin \theta$; the r is a signed distance.

Example: $r = 2$ gives the circle $x^2 + y^2 = 4$.

154 Polar Curves: Common Shapes

Explanation: $r = a$ (circle), $r = a \cos \theta$ (circle through origin), $r = a(1 + \cos \theta)$ (cardioid), $r = a \sin(n \theta)$ (rose with n petals if n odd, $2n$ if even), $r = a \theta$ (spiral).

AP Calc BC Tip: memorize the petal count rule: $r = \sin(n \theta)$ has n petals if n is odd, $2n$ petals if n is even.

Example: $r = 3 \sin(2 \theta)$ has 4 petals; $r = 3 \sin(3 \theta)$ has 3 petals.

155 Slopes in Polar

Explanation: $dy/dx = [dr/d\theta \sin \theta + r \cos \theta] / [dr/d\theta \cos \theta - r \sin \theta]$.

AP Calc BC Tip: derive from $x = r \cos \theta$, $y = r \sin \theta$ and the chain rule.

Example: For $r = 2$ at $\theta = \pi/2$: $dr/d\theta = 0$, so $dy/dx = -\cot \theta = 0$ (horizontal tangent at top).

156 Area in Polar (Revisited)

Explanation: $A = (1/2) \int_a^b r^2 d\theta$; find a , b where petals/curves begin and end.

AP Calc BC Tip: trace the curve as θ sweeps to identify correct bounds (petals often 0 to π/n).

Example: Rose $r = \sin(2 \theta)$: one petal from 0 to $\pi/2$ gives $A = \pi/8$.

157 Intersections of Polar Curves

Explanation: Solve $r_1 = r_2$, but also check the pole ($r = 0$) -- intersections can occur where both pass through the origin.

AP Calc BC Tip: the pole is an intersection even when the equations never equal at the same θ .

Example: $r = 1$ and $r = 2 \cos \theta$ meet at $\theta = \pi/3, 5\pi/3$, AND at the pole ($r = 0$).

158 Arc Length in Polar (Revisited)

Explanation: $L = \int_a^b \sqrt{r^2 + (dr/d\theta)^2} d\theta$.

AP Calc BC Tip: the integrand is speed in polar -- $r^2 + (r')^2$ under the radical.

Example: $r = a$ (circle): $L = \int_0^{2\pi} \sqrt{a^2} d\theta = 2\pi a$.

159 The Distance Traveled in Parametric Motion

Explanation: Total distance = integral of speed = $\int \sqrt{(dx/dt)^2 + (dy/dt)^2} dt$ over the time interval.

AP Calc BC Tip: this equals the parametric arc length -- same integral.

Example: $x = t$, $y = t^2$ on $[0, 1]$: distance = $\int_0^1 \sqrt{1 + 4t^2} dt$.

160 Particle Position from Velocity

Explanation: Position = initial position + integral of velocity: $r(t) = r(t_0) + \int_{t_0}^t v(u) du$.

AP Calc BC Tip: integrate velocity component-wise; add the initial position vector.

Example: $v = \langle 2t, 1 \rangle$ with $r(0) = \langle 0, 0 \rangle$: $r(t) = \langle t^2, t \rangle$.

161 Tangent Lines to Polar Curves at a Point

Explanation: Evaluate $dr/d\theta$ and the coordinates at the given θ , then use the polar slope formula.

AP Calc BC Tip: convert the point to (x, y) first for the tangent line equation.

Example: At $\theta = \pi/2$ for $r = 1 + \cos \theta$: $r = 1$, point $(0, 1)$, slope 0 (horizontal).

Explanation: Even functions in $\cos \theta$ are symmetric about the polar axis; even in $\sin \theta$ symmetric about the $\pi/2$ line.

AP Calc BC Tip: *symmetry halves integration bounds for area.*

Example: $r = \cos(2\theta)$ is symmetric about the polar axis and the vertical line.

Unit 10: Unit 10: Infinite Sequences & Series

~17-18% of the exam. The BC-defining unit: convergence tests and Taylor series.

163 Sequences vs Series

Explanation: A sequence is an ordered list $a(n)$; a series is the SUM of a sequence's terms, sum $a(n)$ from $n = 1$ to ∞ .

AP Calc BC Tip: *sequences have limits; series have sums -- don't conflate them.*

Example: $a(n) = 1/n$ is a sequence; sum $1/n$ is the harmonic series (diverges).

164 Convergence of Sequences

Explanation: A sequence converges if its terms approach a finite limit L as $n \rightarrow \infty$; otherwise it diverges.

AP Calc BC Tip: *use L'Hopital on the continuous version $a(x)$ when $a(n)$ is indeterminate.*

Example: $a(n) = n/(n+1)$ converges to 1; $a(n) = (-1)^n$ diverges (oscillates).

165 The nth-Term (Divergence) Test

Explanation: If $\lim a(n)$ as $n \rightarrow \infty \neq 0$ or does not exist, then sum $a(n)$ DIVERGES. Zero limit gives NO conclusion.

AP Calc BC Tip: *this test can only prove divergence -- a zero limit proves nothing.*

Example: sum $n/(n+1)$ diverges since the terms tend to 1, not 0.

166 Geometric Series

Explanation: sum $a r^{(n-1)}$ converges to $a/(1-r)$ when $|r| < 1$ and diverges when $|r| \geq 1$.

AP Calc BC Tip: *check $|r| < 1$ first; the sum formula applies only when convergent.*

Example: sum $3(1/2)^{(n-1)} = 3/(1 - 1/2) = 6$.

167 The Harmonic & p-Series

Explanation: sum $1/n^p$ converges iff $p > 1$; the harmonic series ($p = 1$) diverges.

AP Calc BC Tip: *p-series is the benchmark for comparison tests.*

Example: sum $1/n^2$ converges; sum $1/\sqrt{n}$ diverges ($p = 1/2$).

168 The Integral Test

Explanation: For positive decreasing f with $f(n) = a(n)$, sum $a(n)$ converges iff integral from 1 to ∞ of $f(x) dx$ converges.

AP Calc BC Tip: *verify positivity and decreasing BEFORE applying the integral test.*

Example: sum $1/n^2$ converges because integral $1/x^2 dx$ converges.

169 Direct Comparison Test

Explanation: If $0 \leq a(n) \leq b(n)$ and sum $b(n)$ converges, then sum $a(n)$ converges; if $a(n) \geq b(n) \geq 0$ and sum $b(n)$ diverges, sum $a(n)$ diverges.

AP Calc BC Tip: *compare to a known benchmark (p-series, geometric).*

Example: sum $1/(n^2 + 1)$ converges by comparison with $1/n^2$.

170 Limit Comparison Test

Explanation: If $\lim a(n)/b(n) = c$ with $0 < c < \infty$, then sum $a(n)$ and sum $b(n)$ converge or diverge together.

AP Calc BC Tip: *use limit comparison when direct comparison's inequality is awkward.*

Example: sum $1/(n^2 - n)$ behaves like $1/n^2$: limit comparison gives convergence.

171 The Alternating Series Test

Explanation: If $a(n) > 0$, decreasing, and $a(n) \rightarrow 0$, then $\sum (-1)^{(n-1)} a(n)$ converges.

AP Calc BC Tip: *verify all three conditions; the terms must decrease to zero.*

Example: $\sum (-1)^{(n-1)}/n$ converges (alternating harmonic).

172 Alternating Series Error Bound

Explanation: For a convergent alternating series, the truncation error is at most the first omitted term: $|S - S(n)| \leq a(n+1)$.

AP Calc BC Tip: *the error bound is the size of the next term -- a free FRQ point.*

Example: $\sum (-1)^{(n-1)}/n$: error after 10 terms $\leq 1/11$.

173 Absolute vs Conditional Convergence

Explanation: $\sum a(n)$ converges absolutely if $\sum |a(n)|$ converges; conditionally if it converges but not absolutely.

AP Calc BC Tip: *absolute convergence implies convergence; conditional series like the alternating harmonic need care.*

Example: $\sum (-1)^{(n-1)}/n$ converges conditionally ($|terms|$ diverge).

174 The Ratio Test

Explanation: If $\lim |a(n+1)/a(n)| = L$: converges absolutely if $L < 1$, diverges if $L > 1$, inconclusive if $L = 1$.

AP Calc BC Tip: *the ratio test is the go-to for factorials and exponentials.*

Example: $\sum n!/n^n$ converges: ratio $\rightarrow 1/e < 1$.

175 The Root Test

Explanation: If $\lim |a(n)|^{1/n} = L$: converges absolutely if $L < 1$, diverges if $L > 1$, inconclusive if $L = 1$.

AP Calc BC Tip: *the root test shines for n th powers like $(something)^n$.*

Example: $\sum (n/(n+1))^n$: root test limit $1/e < 1$, converges.

176 Power Series & Radius of Convergence

Explanation: A power series $\sum c(n)(x - a)^n$ converges on an interval centered at a ; the radius R defines where $|x - a| < R$.

AP Calc BC Tip: *find R with the ratio test, then check the ENDPOINTS separately.*

Example: $\sum x^n/n$: $R = 1$, converges on $[-1, 1)$ (conditional at $x = -1$).

177 The Interval of Convergence

Explanation: Test endpoints individually: plug $x = a \pm R$ into the series and classify convergence.

AP Calc BC Tip: *endpoints are where the ratio test fails -- use other tests there.*

Example: $\sum x^n$ has interval $(-1, 1)$: diverges at both endpoints.

178 Taylor Polynomials

Explanation: The n th Taylor polynomial of f at a : $P(n)(x) = \sum f^{(k)}(a)/k! * (x - a)^k$ for $k = 0..n$.

AP Calc BC Tip: *$f^{(0)}(a) = f(a)$; compute derivatives carefully at the center.*

Example: For $f(x) = e^x$ at 0: $P(3)(x) = 1 + x + x^2/2 + x^3/6$.

179 Taylor & Maclaurin Series

Explanation: A Taylor series at $x = a$: $f(x) = \sum f^{(k)}(a)/k! (x - a)^k$; a Maclaurin series is the Taylor series at $a = 0$.

AP Calc BC Tip: *memorize the core Maclaurin series: e^x , $\sin x$, $\cos x$, $1/(1 - x)$, $\ln(1 + x)$, $\arctan x$.*

Example: $1/(1 - x) = 1 + x + x^2 + x^3 + \dots$ for $|x| < 1$.

180 The Lagrange Error Bound

Explanation: $|R(n)(x)| \leq M |x - a|^{(n+1)}/(n+1)!$ where M bounds $|f^{(n+1)}|$ between a and x .

AP Calc BC Tip: *find M as the max of the next derivative on the interval; this bounds Taylor error.*

Example: For e^x on $[0, 1/2]$, $R(3) \leq e^{(1/2)} (1/2)^4/24$.

181 Taylor Series from Known Series

Explanation: Build new series by substitution, differentiation, or integration of known Maclaurin series.

AP Calc BC Tip: $e^{(-x^2)} = \sum (-1)^k x^{(2k)}/k!$ by substitution; integrate term-by-term for integrals.

Example: $\int e^{(-x^2)} dx = x - x^3/3 + x^5/10 - \dots$ (antiderivative series).

182 Convergence of Taylor Series

Explanation: A Taylor series equals $f(x)$ on its interval of convergence when the remainder tends to 0; otherwise it converges to something else.

AP Calc BC Tip: *the Lagrange bound tending to 0 proves the series represents f .*

Example: The Maclaurin series of $1/(1 - x)$ equals the function only for $|x| < 1$.

What's Next?

You have now reviewed every unit of the AP Calculus BC course, from limits to Taylor series. Go back to the Table of Contents, find the units you marked, and reread just those tip lines. Units 6 and 10 are the heaviest on the exam (roughly 17-18% each) and Unit 9 (parametric/polar/vector) is BC-only material the AB exam never touches -- give them extra passes. Practice every formula until automatic, and drill the convergence tests and Taylor series until you can apply them without hesitation. Two weeks out, do a rapid pass over the entire guide; the day before, review only the tip lines. On the exam, show every step and justify convergence choices explicitly -- partial credit and justification points reward written reasoning.

Pair it with the AP Calculus AB Study Guide (\$19) -- AB material is the first 8 units of BC

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